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DESIGN OF CAM PROFILES FORMED BY ELASTIC ELEMENTS

A THESIS

Presented to

The Faculty of the Graduate Division

by

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DESIGN OF CAM PROFILES

FORMED BY ELASTIC ELEMENTS

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SUMMARY

The elastic band cam is fabricated using a solid boss, a thin elastic band for its profile, and a backing by a matrix material. The band is constrained to have a desired rise, angle of rise, return, and dwell. Its chief advantage is the continuity of the profile which is based on an elastic deflection curve. This curve has continuous derivatives at all points except where forces and moments are applied to the band, and the cam can be designed so the velocity and acceleration are continuous at these points as well. The method of manufacture of the elastic band cam is unique and eliminates the problems inherent to the incremental cutting through a series of precision points used to make cams of conventional design. The elastic band forming the profile has the exact shape it is designed to have since it is the elastic curve that the theoretical profile is based on.

The shape of the elastic band cam is found by finding the equations which define the elastic curve forming the profile. These equations are obtained by locating a portion of a known elastic curve which is congruent with the cam curve. Then the equations defining the cam curve are obtained from the given equations of the known elastic curve. This method is used in lieu of solving the second order non-linear differential equation defining the elastic curve from the relation between the bending moment in and the curvature of the elastic band.

Unlike conventional cams where the follower path is the designed motion, when an elastic band cam is used the cam profile is the fixed

motion. Hence the path of the follower is a function of the type of follower system and the cam profile. Therefore the shape of the path of a follower operating on this cam must be calculated from the cam profile.

The analysis of the elastic band cam is completed with the actual fabrication and testing of some sample cams. The purpose of this portion of the work is to verify that a cam can be fabricated according to the procedures outlined, and that the cams produced in such a manner do correlate with the theoretically predicted cam.

A set of data is included which can be used as an aid in the design of elastic band cams. This is prepared using the theoretical descriptions of the cams and given the acceleration characteristics of many typical cam and follower systems. They allow a designer to predict the dynamic characteristics of an elastic band cam without actually calculating the shape of its profile, and are of great value when selecting a cam for a particular application from the many possibilities.

The elastic band cam can be designed to have dynamic characteristics including a continuous acceleration which, when used with the appropriate follower system, give acceptable follower paths for use in many applications. The cam fabricated to the design specifications has the shape it is designed to have, and the time and cost required to fabricate the first cam is less than that required for the fabrication by conventional methods. Because the elastic band cam is formed from a combination of materials which do not have a high strength or a high elasticity, the cam is best used as a master from which others are copied. Use of the cam as a master retains the advantages of its unique design because these have the greatest effects on the first cam produced, the master.

CHAPTER I

INTRODUCTION

Review of Present Methods

The cam has been used for many years in the design of all types of mechanical items. Its design and production have changed as engineers have developed better types of cams and better methods of production.

An important part of the improvements in cam design involves the development of the cam profiles. One of the simplest types of cam is the circle arc cam. Its profile is composed of circles and straight lines. The ease of design and manufacture are the advantage of this type cam. Unfortunately, its dynamic characteristics are poor. New procedures for further simplifying the design of this profile have been presented by Kahr (1)[*] and Weber (2). Rothbart (3,4) presented procedures which can be used to design classic type cam profiles producing follower motions that are based on constant acceleration, simple harmonic, parabolic, or cycloidal curves. These motions are based on some type of geometric or trigonometric configuration of the displacement diagram so the equations representing their profiles can be written in simple forms. From these equations the first, second, and third derivatives can be calculated. This allows an evaluation of the motion of

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[*]Numbers in parentheses refer to references listed in the Bibliography.

the cam follower. These basic motions have their limitations, so they are not the best solution to all cam problems. They do, however, provide a good base from which to work for much of the continued research in cam design. One such use is the combination of parts of the basic motions to form a better motion. C. N. Neklutin (5) presented a method of combining segments of different trigonometric-type cam motions from which he developed profiles that he called "modified trigonometric-type" of cam profile. This method of combining segments of different types of cam profiles led to the development of the well known modified-trapezoidal cam profile. It is named for the shape of its acceleration curve and is a combination of the parabolic and the cycloidal type motions utilizing the good qualities of each. A useful and complete procedure for its design was developed by Viktoras Bilaisis (6). Methods have been developed by Schmidt (7) and Kloomok and Muffley (8) which combine segments of different cam profiles to produce more suitable cams for specialized uses.

Another type of profile which is useful in cam design is that based upon numerical procedures. The advent of the digital computer has made these methods as practical to use as those based on geometric or trigonometric-type motions. Many of these procedures base the cam curve on a polynomial equation with the constants calculated from the end conditions specified for the desired curve and its derivatives. Kloomok and Muffley (9) used a polynomial with ten constants to develop profiles. A well-known polynomial cam is the "polydyne" cam. The constants of the polynomial are found using boundary conditions for the displacement and its first three derivatives. D. Stoddart (10) has presented a detailed method of design for this type of cam. Johnson (11), and Shafer and Krause (12) used the method of finite differences to develop cam profiles.

Other design methods for cam profiles have also been presented to be used for special purposes. T. Weber, Jr. (13), presented a method of design using filter theory, C. N. Neklutin (14) presented a method whereby the cam was designed based on the acceleration diagram rather than the displacement diagram, and J. H. Carlson (15) based his method on the criterion of a constant load. Methods which predict and eliminate vibration resulting from the cam action were also formulated (16,17).

For many applications where the follower system cannot be considered rigid, the design procedure must include the action of the cam follower as well as the cam. Investigation of the cam and follower system often includes the change in follower motion as a result of the deformation at the point of contact between the cam and follower (18, 19). Also a method was presented by which the motion of the follower was predicted considering the higher harmonics of the basic follower motion (20).

Study of the cam-follower system is of particular interest for automotive cams. In the internal combustion engine the follower system is relatively flexible and the camshaft must run at high speeds as well as through a large range of speeds. The analysis of the cam alone is not sufficient. The valve opening and the engine performance, rather than the actual cam profile, are the basis of cam evaluation. In 1948 W. M. Dudley (21) presented a method of valve cam design which included effects of mass, flexibility, valve spring force, and gas pressure. His method concluded with the modification of a given ideal cam profile to one which, considering all other factors, would give the required follower motion. A similar method of altering the cam profile using the polydyne cam as the basic cam contour was presented by D. Stoddart (22,23,24) in 1952. Markhauser (25) also used the polydyne cam profile as a basis in his work to eliminate oscillations of the valve train at high engine speeds. In some cases the cam profile may not be based on any classical type motion at all, but just the required valve motion (26). This motion is modified to obtain good dynamic characteristics along with good engine operation.

The digital computer has helped in the development of techniques to predict the motion and vibrations of the follower (valve) for given cam contours. Techniques have been developed which assist in evaluating cam profiles considering the follower response (27,28). Vibrations are used as a factor for determining minimum cam size (29,30) for some operations. The prediction of errors in the cam follower system (31), the deflection in the camshaft caused by the torque on the cam (33), and the effects of spring loading to maintain contact with the cam (34) are other factors which have been considered by engineers in their studies of cam and follower systems.

The manufacture of cams presents another problem. Many methods of manufacture of a given profile are available (35). The actual profile of the cam, which is different from the follower path, must be determined. The calculation of the shape of the cam profile depends upon the type of follower system to be used. Simplified mathematical methods have been developed to obtain the coordinate points for a given system (36,37). These may be used in place of graphical layouts. The computer has proven helpful for calculating the coordinates to very high degrees of accuracy (38).

The usual way of producing a cam from design specifications is the method of incremental cutting; that is, the cam is formed by making straight or circular cuts through a finite number of precision points. Then the surface is smoothed between the precision points using a hand or machine approximation. In order to eliminate some of the manufacturing errors resulting from incremental cutting, new manufacturing methods and machinery have been developed. Recent innovations in cam generating equipment are the tape-controlled milling machines and grinders. These machines cut a profile defined by a very large number of coordinate points which are recorded on paper or magnetic tape and fed into the machine. A computer is usually used to produce the high accuracy necessary for the coordinate points to produce tapes accurate enough to utilize these machines to their fullest (39). The advantages of the tape-controlled mills and grinders include their versatility--they do not require a master cam--and the fact that they can maintain a high degree of accuracy (40,41). One disadvantage of the tape-controlled machine is the expense of the equipment involved. This eliminates its use by many small manufacturers whose work might include cam manufacture.

Another innovation in cam manufacture is the use of the electronic template. A machine traces an electrically conductive line on a drawing. Of course the accuracy of such an arrangement is dependent upon the accuracy of the drawing as well as that of the tracer mechanism (42). Here again expensive and specialized equipment, which is not available to many designers, is needed. A machine which will generate many variations of cams from a single master using different linkages between the master and the cutter is also available (43). During the cutting operation this machine duplicates the environmental conditions under which the cam will operate. For complementary cam systems a method has been devised so that the two cams can be machined back to back and therefore will be a matched pair and give improved performance (44).

Most of the previously mentioned methods of manufacture may be utilized to produce a cam from the mathematical or graphical specifications. When cams are to be mass produced, they are almost always produced through a tracing operation which duplicates a master cam. This may be done using a tracer controlled milling machine or cam grinder (45). These use some type of follower on the master which will position the cutter in the correct location with respect to the material from which the cam is being fabricated. The types of followers and systems used to maintain the relation between the follower and cutter vary. The simplest is a mechanical system with a direct linkage from the follower to the cutter. This is simple but usually requires high contact pressures between the master cam and follower. Other systems using electric or hydraulic servo systems between the follower and cutter may be used. There are systems where the follower is not mechanical, but a beam of light. For all of these types very little pressure is required between the master and the follower.

Of course in any manufacturing operation there will be some errors involved. Finite difference methods have been used to predict the dynamic effects on the follower system as a result of machining tolerances and errors (46,47,48). The effects of error from rounding off of theoretical curves in incremental cutting have also been predicted using numerical analysis (49).

Also important to the total manufacturing operation are the inspection and checking of the cam. Many new methods and much new equipment has been developed to keep pace with the new manufacturing methods for cams (50). One new technique uses gauging equipment which utilizes a digital voltmeter to display the values of the coordinate points of the cam contour (51). It provides a very fast yet accurate measurement at precision points spaced as close as 0.30 degrees around the cam. Another precision checking set up which includes a checking device which will measure to radial accuracy of 0.00005 inch is described by J. H. Nourse (52). The data from this instrument is recorded on tape and then fed to a computer programed to evaluate the cam by comparing it to a theoretical model. Both of these methods have only limited applications because of the expense of the equipment involved and because the accuracy of the measurements is normally needed only for the production of a very precise master cam.

The last factor that should be considered here is the wear of the cam and follower. This problem does not affect the dynamics of the cam follower system in the design stage. It is related more to the metallurgical requirements of the cam and follower and the specification of the correct lubricants for the system (53,54,55). Though wear on the cam surface cannot be eliminated completely, preferably the cam follower system is designed so that the follower wears more easily than the cam. In most systems the follower can be adjusted to compensate for wear without major modifications, and is the less expensive to replace if this is necessary. Therefore, much of the discussions of wear of cam systems involves the follower and not the cam.

Cams Using Elastic Curves in Their Design

Previous Work

The idea of using an elastic band to help generate a cam profile has been investigated only in a very limited manner. F. L. Borun (56) in Russia designed a cam using the elastic curve of a deformed steel band as a profile to be traced with a special cam copying fixture. K. Lindner (57) in Germany used an elastic band as a mold to produce a plastic master cam.

Borun outlined a method of basing a cam profile on an elastic curve. He derived the equations for only one configuration of elastic curve, the rise equal to one-half the run. This derivation assumed the curve to be in the form of a linear displacement diagram. No consideration was given to the effects on the elastic curve because the actual band is not in the shape of the displacement diagram, but wrapped around to form the shape of a cam. He prepared velocity and acceleration curves from the displacement diagram and compared the maximum values of the acceleration and the dynamic load factor,

$$\left[\frac{d^2y}{dx^2}\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\right] ,$$

of conventional cam profiles, such as constant acceleration, constant d^3y/dx^3, and modified trapezoidal acceleration with the elastic curve. Although elastic band curves compared well with the conventional cam contours under the conditions given by Borun, these conditions gave false impressions because the comparison values are based on the displacement diagram elastic curve and not the true shape. Also Borun assumed that the acceleration curve would not be discontinuous even though the drawing of the acceleration curve seemed to have a finite jump at the point of initial rise. He attributes this seeming discrepancy to the clamping of the band in a copying fixture, which he says will cause the acceleration curve to be continuous.

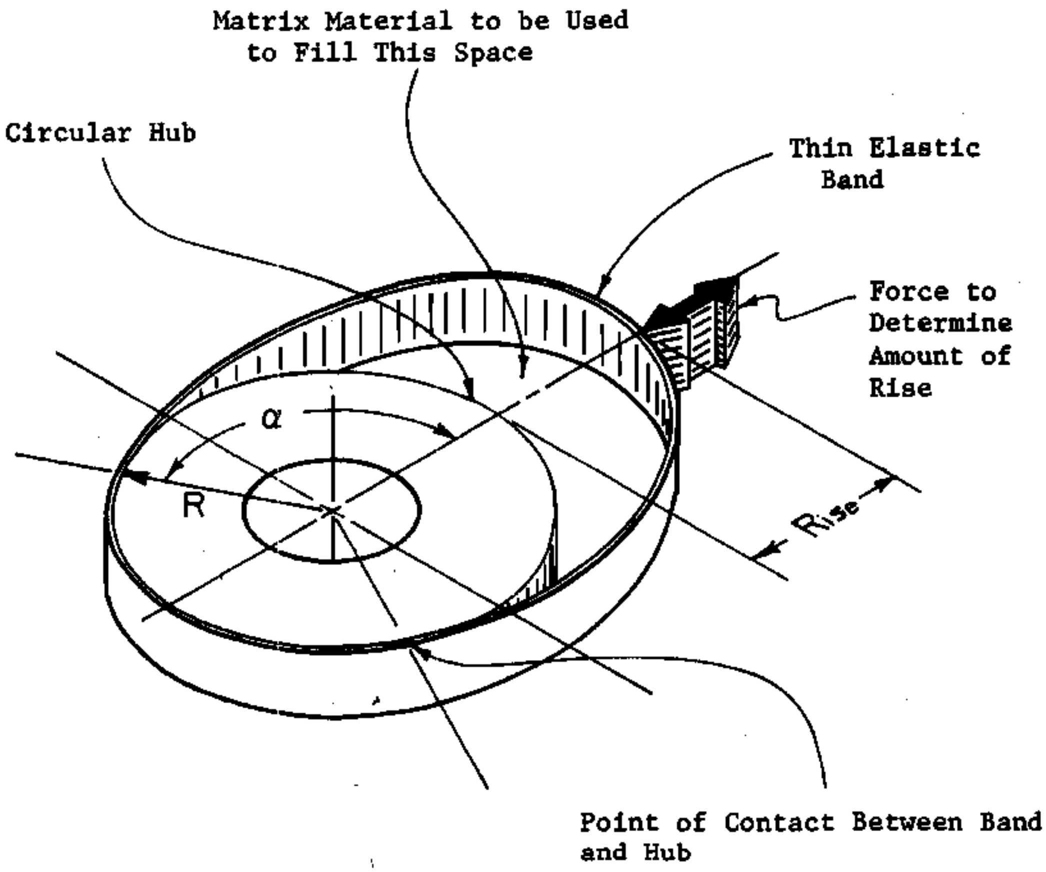
In his paper, K. Lindner discussed the production of a master cam using an elastic band as a mold for a plastic master cam. His purpose was to duplicate existing cam profiles. The mold was formed by clamping an elastic band at a large number of precision points so that the mold would have the same profile as the cam to be duplicated; therefore, the need for incremental cutting to produce the master cam would be eliminated. The elastic curve was only used to approximate the cam profile between precision points and not as a basis for the profile itself. He did no analytical work to predict the curve between the precision points.

The Elastic Band Cam

The two basic problems associated with cams are the profile design and the cam production. The profile should have acceptable characteristics with respect to both the function which it will serve and the dynamics of the system, and the production requires suitable accuracy to design specifications with the required accuracy.

The elastic band cam provides a solution to both of these problems through its basic structure. Its formation from a continuous elastic band provides a method of fabrication producing the exact design contour and this contour can be designed to have the necessary dynamic characteristics. This cam is fabricated using a solid boss, a thin elastic band for its profile, and a backing by matrix material (Figure 1). The angle of rise always equals the angle of return and therefore it is symmetric. Its advantages stem from the fact that its profile consists of a continuous elastic band. Its shape is that of an elastic deflection curve. Therefore it has continuous derivatives at all points between constraining forces. When an external point force is applied to the band, the shape of the curve will change and only the first and second derivatives will be continuous at that point. At points where both an external force and moment are applied to the band, only the first derivative is continuous. Referring again to Figure 1, note that the cam profile has a continuous displacement with continuous velocity, acceleration, and all higher derivatives at all points except the points where the band makes contact with the hub and at the toe of the cam. At the toe the acceleration is continuous, and at the points of contact the acceleration can be made continuous for special configurations of the cam. A cam is made to fit the design specifications by constraining a band of the correct length so that it has the desired angles of rise and return and the correct measure of rise. The shape of the profile is determined by the shape of the free elastic curve of the band.

An important quality of the elastic band cam is the ease of fabrication. Most cams are made from design specifications using the



α = Angle of Rise

R = Radius of Base Circle

Figure 1. The Elastic Band Cam

method of incremental cutting. In contrast, the elastic band cam is produced by constraining the elastic band according to previously calculated conditions necessary to produce the required elastic curve. Then the void within the cam is filled with a matrix material. This produces directly a smooth profile which is exactly the profile as predicted by the mathematical representation of the elastic curve. There is no approximation process involved.

CHAPTER II

OBJECTIVE

Justification for the Work

The initial step when undertaking the study of the elastic band cam was to determine if useful results could be expected from this work. The answer to this question can be found by referring back to the basic definition of the cam and its expected properties as given in Chapter I. The cam profile is formed from a continuous steel band which will have the shape of an elastic deflection curve over the rise and return portions. The derivatives of this curve are continuous except at points where there is an external constraint on the band, that is, at the points of contact with the hub and at the toe of the cam. At the toe only the velocity and acceleration are continuous, and at the points of contact with the hub, only the velocity is necessarily continuous. Fortunately, the discontinuities in the acceleration can be eliminated by designing a special case of the cam where the curvature of the band equals the curvature of the hub at the point of contact between the two. These general characteristics, such as a continuous acceleration, which the elastic band cam can possess are characteristics that any cam with good dynamic qualities also possess. Further study to determine more about the characteristics of elastic band cams could be expected to show that they would be of use for practical applications.

The theoretical profile is not the only quality of the elastic band cam which might prove an advancement in cam design. The elastic band cam is fabricated by a process entirely unlike any conventional method. The cam is formed by constraining an elastic band according to the specifications which define the cam. In this way the cam is produced using the same principles for its fabrication as were used as the basis of its profile: the elastic deflection curve. Therefore, the actual cam profile should be the exact shape of the design profile. Also, the elastic band forms a continuous, mechanically smooth surface on the cam. This method of fabrication is an innovation which may prove very useful for cam manufacture, and therefore further enhances the desirability of continued study of this cam.

Procedures

The study of the elastic band cam involves two basic parts: (1) the analytical predictions of the cam profile and follower paths, and (2) the experimental work including the fabrication and testing of sample elastic band cams. The first step in the analytical evaluation of the cams is to obtain the shapes of the elastic curves forming the cams and thereby describe the cam profile. One general method of finding the shape of a deflected elastic member is to establish the second order non-linear differential equation relating the curvature to the moment at any point on the member from the Bernoulli-Euler law. The differential equation then must be solved to obtain the equations for the shape of the curve. An alternate method is to find an elastic curve which is defined by a set of known equations and part of whose shape is the same as the shape of the elastic cam curve. The equations defining the cam curve are then obtained from the equations of the known curve. This method, based on the principle of elastic similarity, is used in this study.

First, the equations for a set of elastic curves known as elasticas are developed according to work by R. Frisch-Fay (58). The cam curves can all be matched to a portion of one of these elasticas and the equations for the cam curve obtained from the equations of the elasticas. The coordinate system in which the cam curves are initially described is not appropriate for use as the coordinate system of the cam. The equations are transformed to a polar coordinate system with the origin at the center of the cam to simplify the evaluation of the profile.

The evaluation of the cam must include the action of a follower operating on the cam, not just the profile of the cam. The elastic band cam differs from conventional cams in this respect. In conventional cam design, the path of the follower is fixed by the design and a cam profile developed from this follower path, so that the action of the specified type of follower on the cam matches the designed follower path. For the elastic band cam the shape of the cam is fixed and the follower paths differ for each type of follower. Therefore, each elastic band cam must be evaluated with respect to the follower path it produces with given followers. For this purpose procedures are developed which calculate the path of a given follower from the profile of an elastic band cam. The evaluation of the cams involves the presentation and comparison of maximum accelerations, changes in acceleration or jerks, and higher harmonics of the accelerations. Computer programs are used to calculate the cam profile, the follower paths, and their derivatives and harmonics.

The experimental portion of the analysis includes the fabrication and testing of three sample elastic band cams. The purpose is to verify that a cam can be fabricated according to the procedure outlined, and that the cams produced in such a manner do correlate with the analytic data calculated. A fixture to facilitate the cam manufacture and a cam testing jig were designed and built. The holding fixture is required to apply the required constraints on an elastic band while the matrix material sets and fixes the shape of the cam. The testing fixture is designed to measure the camshaft displacement, the cam follower displacement, and the follower acceleration. Plots of the follower displacement versus camshaft displacement and follower acceleration versus camshaft displacement are produced from measurements from the test jig. These plots are compared with the corresponding plots compiled from the analytical data to show that the predicted follower path is the same as the path of a follower on a sample cam.

Results to be Obtained

With the procedures for analyzing elastic band cams developed and verified experimentally, the next step is to obtain and present a set of data that will be an aid in the design of elastic band cams. This set of data allows a designer to predict some of the characteristics of a cam without going through an involved series of calculations. The important output from any cam is the follower path, but for a particular elastic band cam, the follower output path will vary with the type and dimensions of the follower system. To obtain the acceleration characteristics of the follower paths on an elastic band cam, first a set of charts showing the acceleration characteristics of the cam profiles is presented. Then a second set of charts is presented which shows a comparison of the cam profile with the follower path for typical follower systems. Using these two sets of charts, the acceleration of a follower operating on any elastic band cam can be predicted. Additional information about some typical elastic band cams and followers is obtained through a harmonic analysis of their acceleration curves. These constituent harmonics are presented so that the changes in the harmonics due to variations of the parameters of the cam may be noted.

This set of data allows a designer to select a suitable elastic band cam from a number which might fit his basic specifications by allowing him to predict the characteristics of the possible choices. He may need to calculate the complete characteristics of the cam he selects, but he need only do this for one cam and not all the possible choices.

In summary, the objective of the thesis is to derive expressions which will analytically describe the shape of elastic band cams and the paths of followers operating on the cams. These are experimentally verified along with the feasibility of manufacture. Finally, using the expressions derived, a set of data is calculated which can be used by a designer to predict the characteristics of an elastic band cam and the follower system.

CHAPTER III

THEORETICAL ANALYSIS OF ELASTIC BAND CAM PROFILES

The key to the analysis of the profiles of the elastic band cam lies in the formulation of a mathematical expression for the curve of the elastic band forming the contour. This solution is basically the analysis of a thin elastic beam loaded in such a manner as to make its deflections large.

Elastic Curve for a Beam with a Large Deflection

Investigation of the deflections of a loaded beam may begin with the Bernoulli-Euler law, which states that the bending moment at any point on the beam is proportional to the change in curvature caused by the action of the load. The basic formula

$$\frac{1}{r} = \frac{M}{EI} = \frac{d\phi}{ds} \tag{3-1}$$

is immediately applicable when the equation of the deflection curve is given in the intrinsic form $S = f(\phi)$ where s is measured along the length of the arc and ϕ is the slope at s. The curvature is expressed in rectangular coordinates as

$$\frac{1}{r} = - \frac{d^2y/dx^2}{[1 + (dy/dx)^2]^{3/2}} \tag{3-2}$$

The negative sign results from the convention of considering downward deflections positive. Combination of Equations (3-1) and (3-2) results in a second order nonlinear differential equation from which can be derived the equation of the deflection curve:

$$\frac{M}{EI} = -\frac{d^2y/dx^2}{[1 + (dy/dx)^2]^{2/3}} \tag{3-3}$$

where M is a function of y.

In much engineering work where the load on the beam will cause only a small deflection, the higher order differential $(dy/dx)^2$ is often neglected, producing a linear solution for the deflection curve. But in the case of a slender beam, such as the elastic band here considered, the deflections will be large and the slope is not negligible. Hence, the nonlinear form must be kept intact.

The buckled shape of a vertical thin column under vertical load (Figure 2), which is a simple case, has been presented in equation form by R. Frisch-Fay (58). He gives equations for the x and y coordinates of any point on the column and the length of the arc measured along the column to any point, such as B. These are:

$$x = \frac{1}{k}[2E(p,\phi) - F(p,\phi)]$$

$$y = \frac{2p}{k}[2\ p\ \cos\phi] \tag{3-4}$$

$$s = \frac{1}{k}\ F(p,\phi)$$

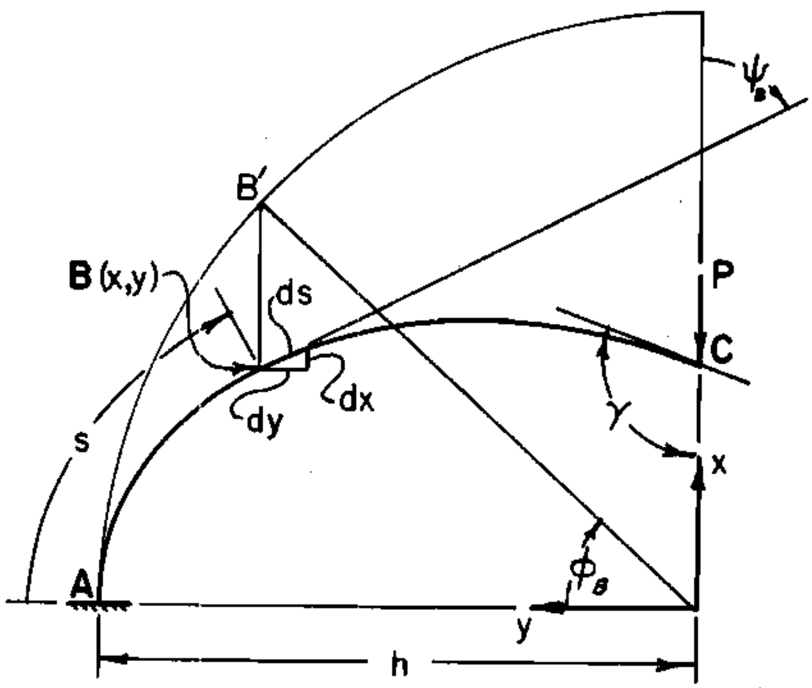


Figure 2. Vertical Thin Column Under Vertical Load

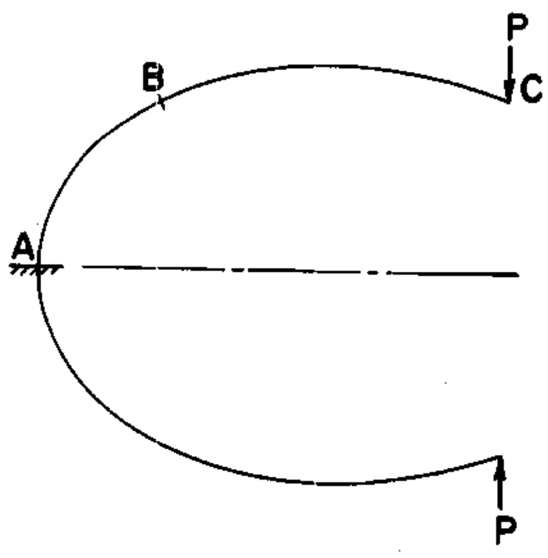


Figure 3. Reflecting a Basic Strut About the Ground Line

where he defined $k = \left(\frac{P}{EI}\right)^{1/2}$ and h as the total horizontal deflection. Then p is defined as a proportionality factor between the two quantities:

$$p = \frac{1}{2}(h\ k), \quad 0 < p^2 < 1.$$

The angle ψ is defined as the angle between the tangent to the beam and the x axis. Then the angle ϕ is defined from ψ and p and substituted into the equations so that they can be expressed in terms of elliptic integrals:

$$\phi = \sin^{-1}\left[\frac{\sin\psi/2}{p}\right].$$

Also used in the equations are:

P = Vertical Force on the Beam.

E = Young's Modulus of Elasticity.

I = Moment of Inertia of the Beam.

In these equations $E(p,\phi)$ and $F(p,\phi)$ represent the Jacobian elliptic integrals. They are expressed in Legendre's standard form for the first kind as:

$$F(p,\phi) = \int_0^{\phi} \frac{d\phi}{(1 - p^2\sin^2\phi)^{1/2}}$$

and for the second kind as:

$$E(p,\phi) = \int_0^{\phi} (1 - p^2\sin^2\phi)^{1/2} d\phi$$

The complete elliptic integrals will also be used: They are special cases of elliptic integrals when the upper limit ϕ equals $\frac{\pi}{2}$. A complete elliptic integral of the first kind is:

$$K(p) = \int_0^{\pi/2} \frac{d\phi}{(1 - p^2\sin^2\phi)^{1/2}}$$

and a complete elliptic integral of the second kind is:

$$E_1(p) = \int_0^{\pi/2} (1 - p^2\sin^2\phi)^{1/2}d\phi$$

Frisch-Fay denotes the modulus of the elliptic integrals as p. This is not the same as the section modulus of the beam, EI. To utilize this solution the modulus p must be found using the geometric configurations of the column and assuming it does not change in length during bending. The vertical column configuration will be referred to as a basic strut.

If the figure of Figure 2 is reflected about the fixed ground line, the symmetric form Figure 3 results. Here the ground member is eliminated. This curve is twice the length of the original curve AC and is acted on by equal and opposite forces P at the two ends. If a series of such shapes are added together by joining them at the ends so that the end forces will be equal and opposite, a shape such as is shown in Figure 4(a) is formed. If the deflected shape of the original vertical column were such that the free end deflected below the level of the ground line, a final shape similar to the shape in Figure 4(b)

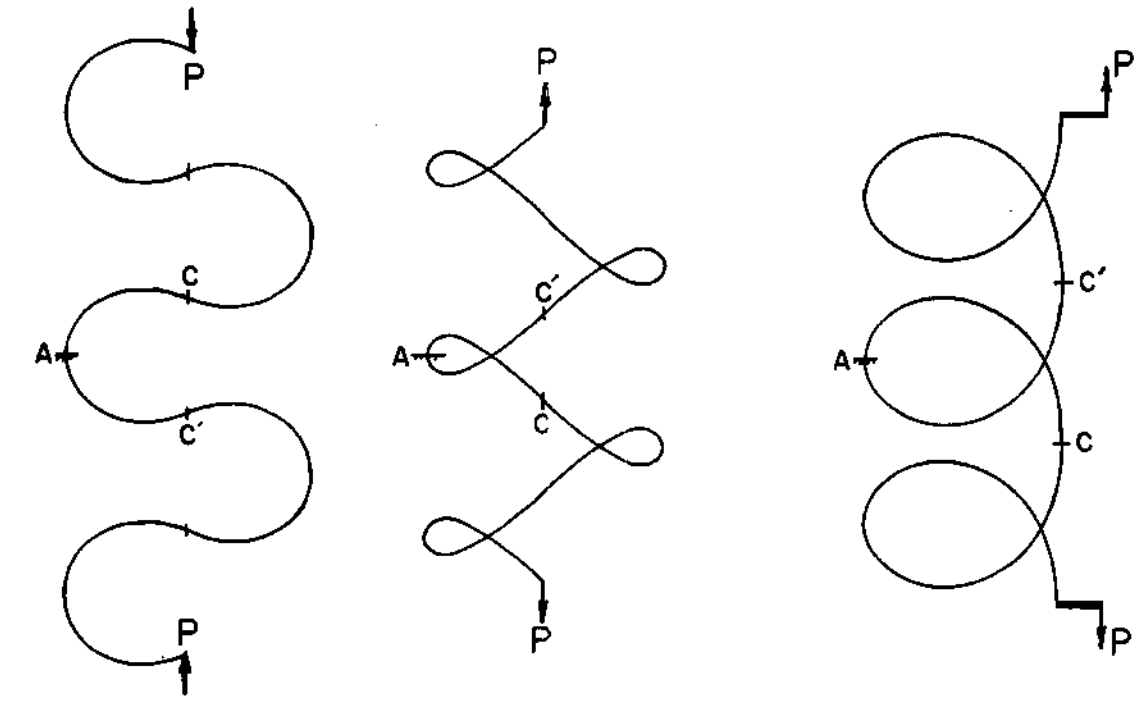


Figure 4. Elastic Curves (Elasticas) Formed from a Series of Basic Struts

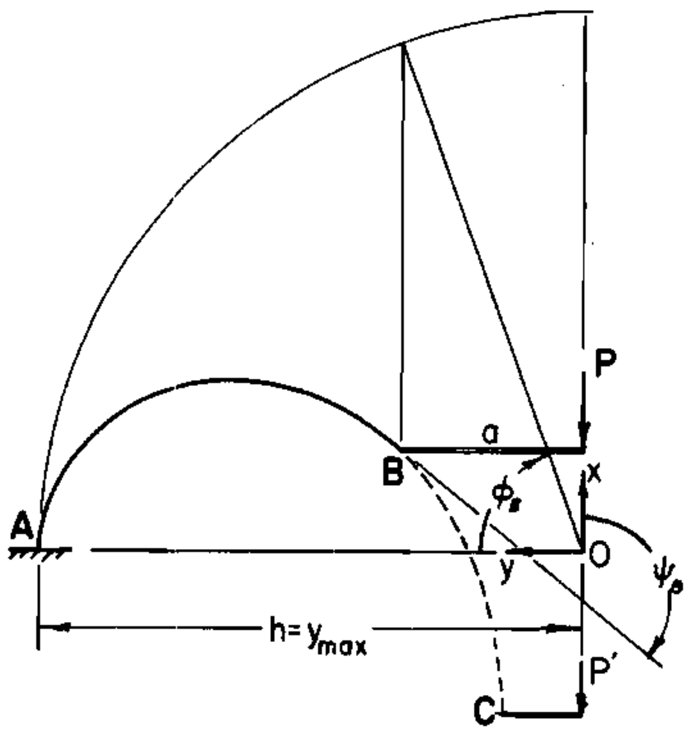


Figure 5. A Vertical Column Under Vertical Load Applied Through a Rigid Lever Producing a Moment Pa at B

is formed. These elastic curves are called "Undulating Elasticas." Note that the points C and C' where the U-shapes were joined are points of contraflexure, i.e. points at which the curvature is zero.

A different shape will be formed by a vertical thin column when it is buckled by a force P applied through a constant moment arm a, as is shown in Figure 5. The direction of the force P is always vertical, and the moment applied by it to the end-point of the column is always M = Pa. Also the line of action of P does not intersect a continuation of the elastic curve. This second column configuration is also referred to as a basic strut.

Expressions for this second basic shape are also given by Frisch-Fay and are as follows:

$$x = \frac{2}{pk}[E(p,\psi/2) - (1 - \frac{p^2}{2}) F(p,\psi/2)]$$

$$y = \frac{2}{pk} \cos\phi \tag{3-5}$$

$$s = \frac{p}{k} F(p,\psi/2)$$

where:

$$k = \left(\frac{P}{EI}\right)^{1/2}$$

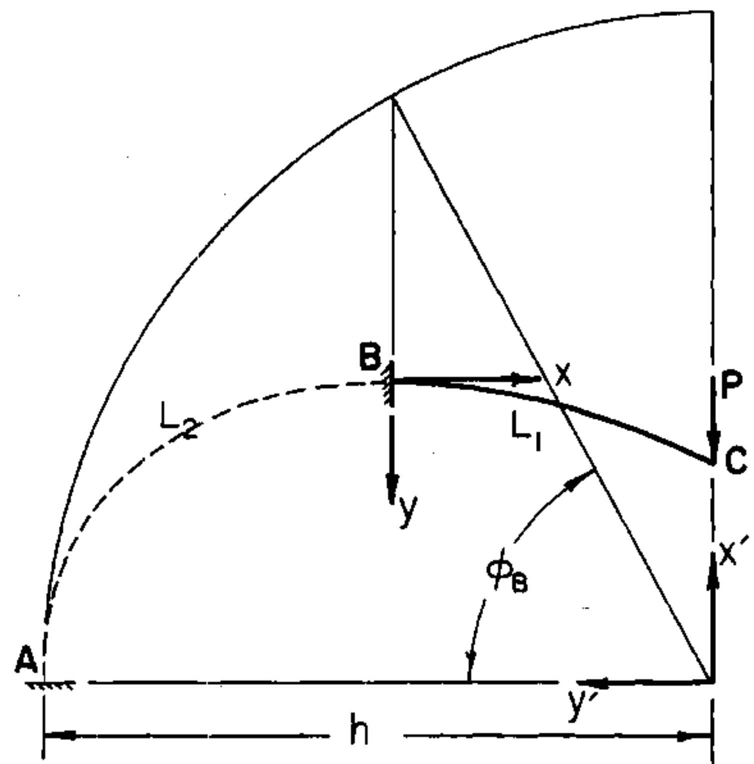
$$p = \frac{2}{k\ h}, \quad (0 < p^2 < 1)$$

$$\phi = \sin^{-1}[p \sin(\psi/2)]$$

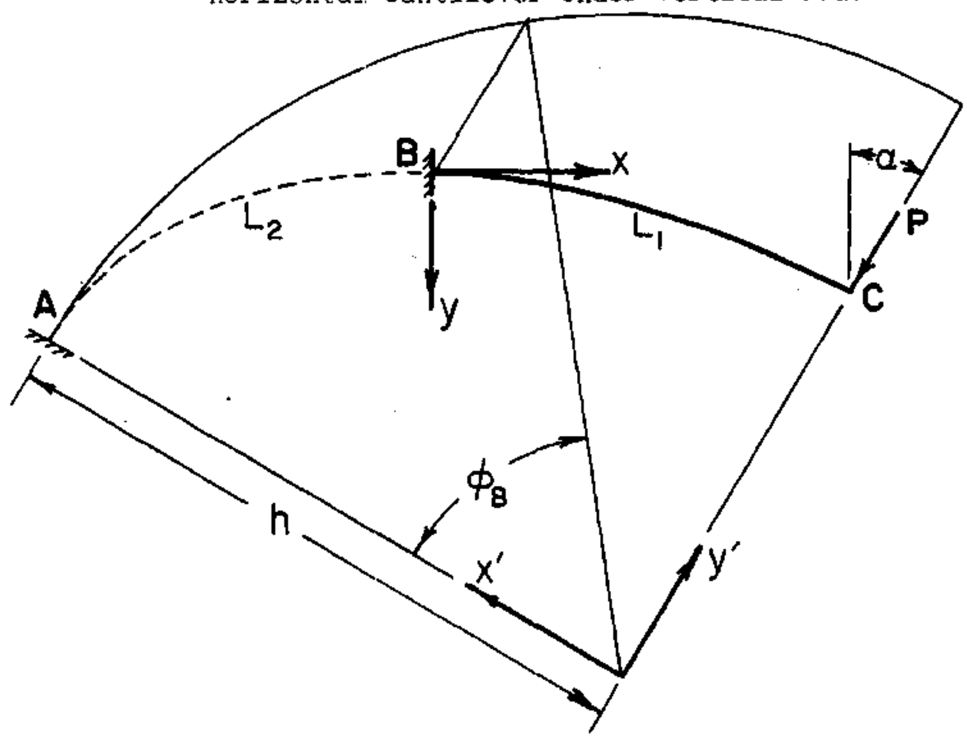
If such a shape as is in Figure 5 is reflected about the ground line, a new U-shape results; and if a series of such shapes are added together, in the same way as it was previously discussed in forming the elasticas of Figure 4(a), (b), so now a new shape of elastica is formed. This appears in Figure 4(c) and is called the "Nodal Elastica." It has only one node (or crossing point) per wavelength. It may also be noted that it has no points of zero curvature.

By the principle of elastic similarity originally enunciated by Euler and further demonstrated by Frisch-Fay (59), an elastica or any other elastic curve can be divided into sections, and if the same force and moment are applied to the ends of the sections as were applied by the continuous curve, each section retains the shape it had when it was a part of the total curve. Therefore the shape of an elastic curve with a given length and section modulus, EI, is a function of the forces and moments on the ends. That is, two elastic members having the same length and section modulus will have the same shape if their end conditions are the same.

In Figure 6 two simple examples show how a cantilever beam can be considered a part of the elastica utilizing the principle of elastic similarity. In these examples the cantilever is fitted to a portion of the elastica so that the load on the beam and the load shaping the elastica coincide. This is at the point of contraflexure of the undulating elastica. The other end of the beam is oriented so that its tangent is the same as the tangent of the elastica at that point. Then assuming that the portion of the elastica and the beam have the same length and the same section modulus, the shape of the elastica



Horizontal Cantilever Under Vertical Load



Horizontal Cantilever Under Inclined Load

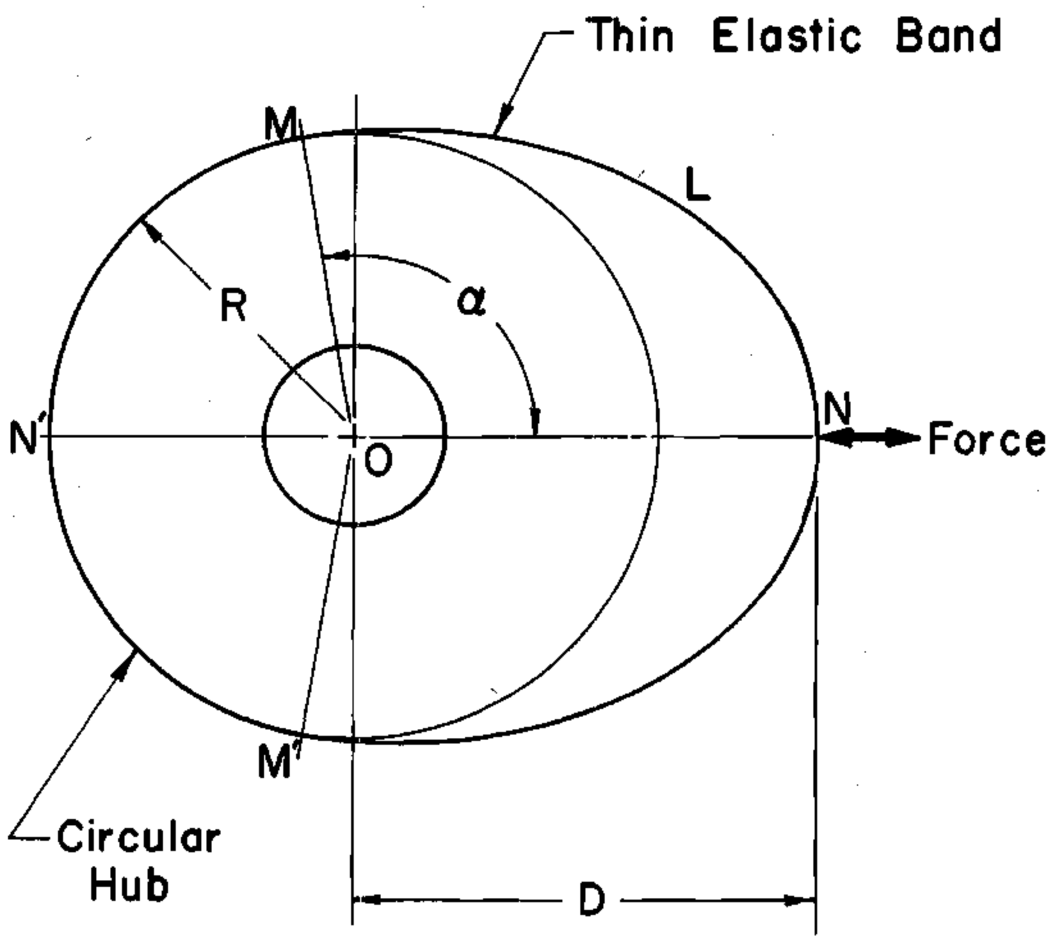
Figure 6. Illustrations of the Principle of Elastic Similarity

and the beam must be the same. By transforming the equations of the portion of the elastica given in the x,y system to the x',y' coordinate system of the beam, the equations describing the elastic deflection curve of the beam are obtained.

The Elastic Curve of the Cam Profile

The principle of elastic similarity can now be applied to the elastic curve of the steel band which forms the profile of any cam under study. The first step in the calculations is to relate the elastic curve of the cam to a part of an elastica so that the end conditions are satisfied, thereby insuring similarity. This is done when the values of the parameters defining the elastica and the exact portion of that elastica that the cam curve fits are found. Secondly, the equations for the elastic curve can always be derived from the equations of the elastica by a simple coordinate transformations.

An important question which must be considered at this point is: Will a fit between any given cam curve and an elastica exist? An elastica is formed by applying a force or both a force and a moment to the ends of an elastic member of given length and section modulus. Because of the infinite number of values of each of these parameters, the force, moment, length, and EI, it is safe to assume that an elastica with a segment having any desired combination of length, section modulus, and forces and moments on the ends can be obtained. Therefore, an elastica will exist which will have a portion to match any physically possible elastic curve.



Parameters:

R = Radius of Base Circle

α = Angle of Rise

D = Distance From Center to Maximum Rise

L = Total Length of Elastic Band

Figure 7. Parameters for Elastic Band Cam

As described in Chapter I the elastic band cam is formed when an elastic band of length L is wrapped around a hub of radius R, leaving the hub so as to give an angle of rise equal to α. It is constrained at the toe by a force so that the distance from the center of the hub to the toe is equal to D (Figure 7). The band completely circumscribes the cam starting and ending at the point N'. The portions from M to N and from N to M' have the shape of a free elastic curve. The cam is always symmetric about the line NN'. Therefore, the shape of the cam profile can be determined if the shape of the elastic curve from M to N is determined

Fitting the Cam Curve to an Elastica

The shape of the elastic curve forming the profile of a cam is found by fitting this curve to an elastica. In order to fit one elastic curve to a portion of another, it must be shown that the sections in question are congruent. Since the shape of the cam curve is not known, the only criteria to determine the particular elastica and what portion of that elastica fits the cam curve are: The position and orientation of the ends, the length, and the section modulus.

The principle of elastic similarity is utilized with this information to fit the cam curve to an elastica. From this principle it is known that two elastic members will be congruent if they have the same length and section modulus and have the same end conditions, that is, the same forces and moments applied at their ends. The forces and moments on the ends of an elastic member will cause a particular orientation of one end with respect to the other. Therefore, if two members of the same length and section modulus run between the same two points, and the orientation between their respective ends is the same, their shapes between the two points are exactly the same.

Now setting up these end conditions, if the two halves of the cam curve which connect at the point of maximum rise are to be formed from a continuous band, constrained by at most a point force, they must have the same tangent and equal curvature at that point. Since the cam is symmetric, this tangent must be perpendicular to the line of symmetry of the cam and the curvature is necessarily the same. At the point where the band meets the base circle, it usually needs to be acted on by both a force and a moment resulting in a fixed end condition. Therefore, the sections of the band on each side of this point need have only a common tangent if they are to be formed from a continuous band. The end points on the band are also a fixed distance from each other. This distance is determined by the lengths R and D and the angle α between them.

On an elastica there are points of maximum curvature occurring at the midpoint of each loop and points of minimum curvature occurring midway between two loops. The curve between a point of maximum curvature and a point of minimum curvature has the form of a basic strut, the basic shape which is used to construct the elasticas. The equations which describe the elastica are the equations for each of these sections. These equations are defined in the x,y coordinate system associated with the basic strut (see Figures 2 and 5). The equations for each section are the same with respect to each coordinate system, but the orientation of the coordinate systems differ. Therefore, to fit the cam curve to an elastica, it must be known if the cam curve

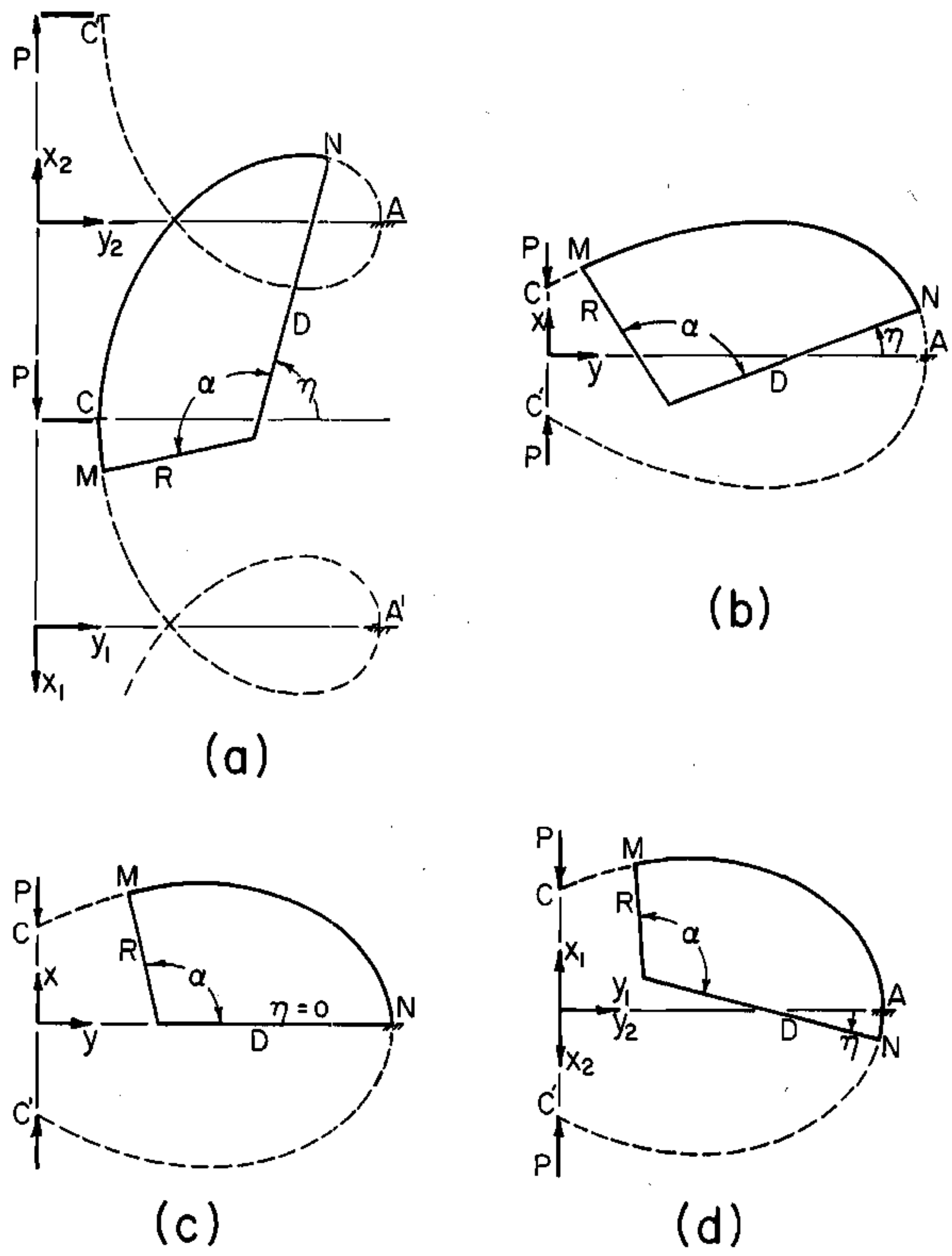


Figure 8. Configurations of the Elastic Cam Curve with the Elasticas

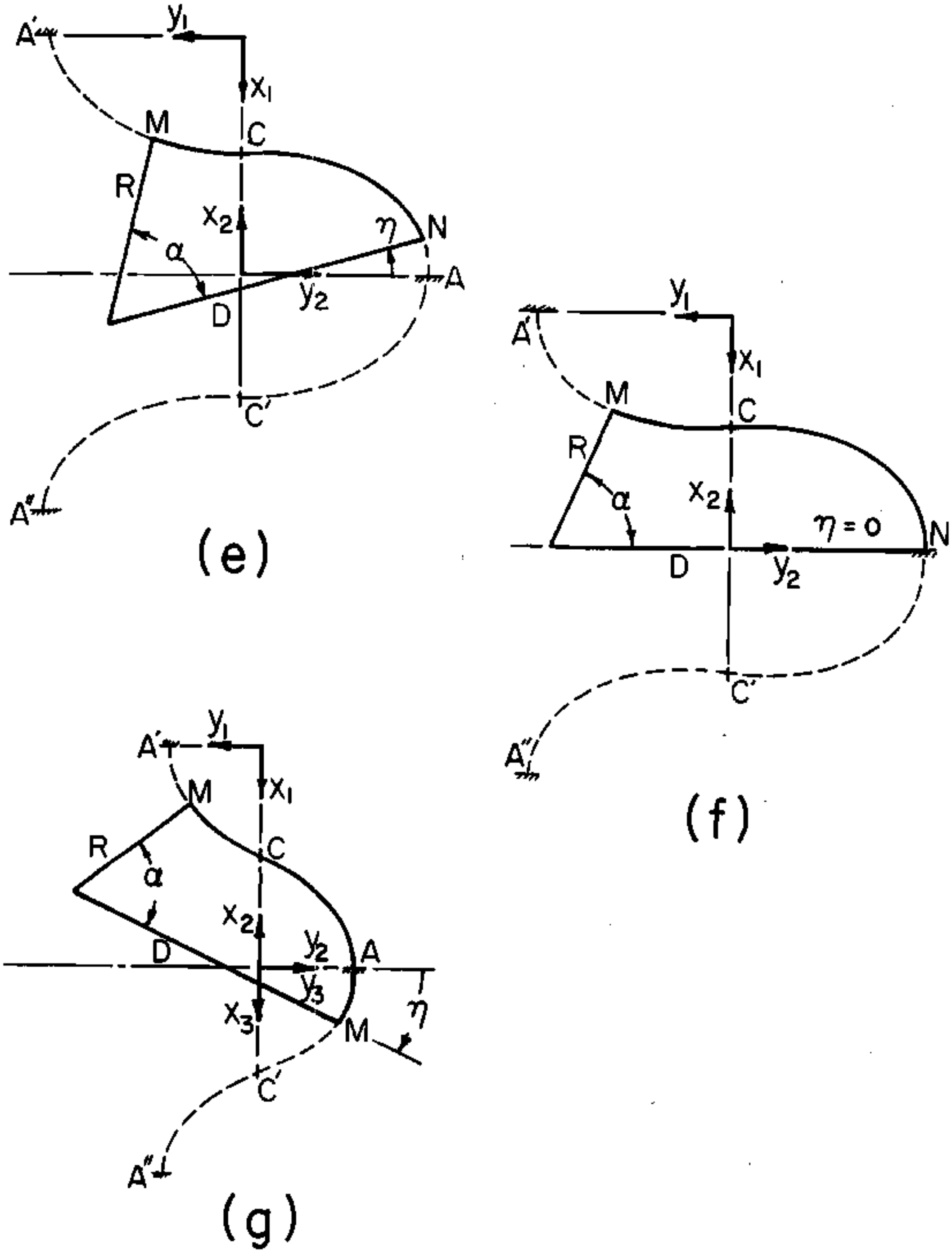


Figure 9. Configurations of Elastic Cam Curves Containing Point of Contraflexure with the Elastica

encompasses only one or more than one of these sections. There are seven configurations of fits between cam curves and elasticas that must be considered (Figures 8 and 9). There are still others possible, but they will produce curves too complicated to be of use as cam profiles and therefore are not of interest for the study of the elastic band cam. In Figure 8(a), (b), (c), and (d) are a progression of fits around a loop of an elastica. For configurations (b), (c), and (d) this may be the loop of either of the elasticas, but configuration (a) fits the nodal elastica only because the two consecutive loops of the elastica must be on the same side of the line of action of the force P. Configurations (e), (f), and (g) (Figure 9) are also a progression of fits around a loop of an elastica. For these three configurations the fit must be with an undulating elastica because the cam curve always includes a point of contraflexure. The parameters defining a particular cam determine which of the configurations the cam curve matches. Note that in Figure 8(a), (d), and Figure 9(e), (f), (g), the fit between the cam curve and an elastica includes portions from more than one of the basic strut sections. Therefore, more than one x,y coordinate system is included for these configurations. Discrimination among the systems in such cases is noted by subscripts on the coordinates. The configurations of fits shown in Figures 8 and 9 will be referred to by the letter associated with them in these figures.

Progressions of the fit of the cam curve with an elastica around a loop as shown in Figures 8 and 9 result when the length of the cam curve is varied while the other cam parameters, R, D, and α, remain constant. The minimum length of the band produces cam curves that fit an elastica according to configuration (a) or (e). As the length of the band increases the angle η, defined as the angle from the centerline of the loop of the elastica to the cam centerline, decreases. More complicated progressions composed of parts of each of the progressions illustrated in Figures 8 and 9 are also a possibility. Such a progression may result when a cam curve defined by a given set of parameters R, D, and α, and the minimum length band fits an elastica according to configuration (a). But as the length of the band increases, a length is reached where the elastic curve must take on a reverse curvature in order to form the specified cam. Therefore, the configuration of the fit between the cam curve and the elastica must change from the type shown in Figure 8 to a fit of the type shown in Figure 9. For this progression the angle η must still decrease continuously as the length of the band increases, just as it did for the simple progression. Once a cam curve has a reverse curvature, it will continue to include a reverse curvature as the length of the band increases.

There are two special cases which occur during a progression. The first is the case where the curvature of the elastic band at the point of contact with the base circle is equal to the curvature of the base circle. This is a very important situation because the resulting cam has a continuous acceleration. This case occurs only when the length of the band is the minimum length and when the center of curvature of the band at the point of contact with the base circle coincides with the cam center. This cam curve fits an elastica according to configuration (a). Although configuration (e) is also a configuration for the minimum length of the band, it can never form this special case because its center of curvature at the point of contact with the base circle is on the opposite side of the band from the center of the base circle, and therefore the two centers can never coincide. There is only one length of band for any given set of parameters R, D, and α, which will form this special case of the cam.

The other special case is formed when no constraint is required at the toe of the cam to give the required rise. Therefore, the cam curve will be a free elastic curve between the points of contact with the base circle. The entire cam curve, not just one half of it, must fit a single elastica. This requires that the center line of the loop of the elastica coincide with the line of symmetry of the cam. This condition is met in either configuration (c) or (f). For any set of parameters R, D, and α, there is only one length of band which will produce a cam according to this special case.

Calculations of the Parameters Defining the Fit Between a Cam Curve and an Elastica

To find the equation of the cam curve we must first determine exactly which elastica and what portion of that elastica the cam curve fits. To do this, the values of the parameters defining the elastica and those locating the correct portion of that elastica, must be found. If we refer back to Equations (3-4) and (3-5), it can be noted that x and y are functions of p, k, and ψ. Therefore, for known values of the two parameters p and k, together with the type of elastica, either undulating or nodal, the elastica is completely defined by the parametric equations for x and y as a function of ψ. The location on the elastica is defined by ψ. To define the end points of the cam curve M and N on the elastica, the values of ψ_M and ψ_N must be found.

Therefore, the parameters for which values must be found in order to define the fit between a cam curve and an elastica are: p, k, ψ_M, and ψ_N, and the type of elastica. To establish the values of these parameters the configuration of the fit between the cam curve and the elastica, i.e. which one of the parts of Figures 8 and 9 represent the fit, must first be decided since the method of determining the values differs for each configuration. The methods whereby the values of parameters p, k, ψ_M, and ψ_N, and the type of elastica can be established for each of the seven configurations are presented in the next seven sections of this chapter. For each case the cam curve is defined by the parameters R, D, α, and L'. L' is the length of the portion of the cam profile from M to N (Figure 7). This can be calculated from the value of the total length of the band L using the equation:

$$L' = \frac{L}{2} - \frac{180 - \alpha}{180} (\pi R) \qquad (3\text{-}6)$$

<u>Cam Curve Configuration (a)</u>. This configuration fits the cam curve with a portion of the nodal elastica (Figure 8 and 10). It will encompass parts of two basic strut sections of the elastica so a separate coordinate system must be used to define each section, x_1, y_1, and x_2, y_2. Given the values of the cam parameters R, D, α and L', we must find the values of p, k, ψ_M, and ψ_N to establish the elastica to which the cam curve is fitted and portion of that elastica used. The values

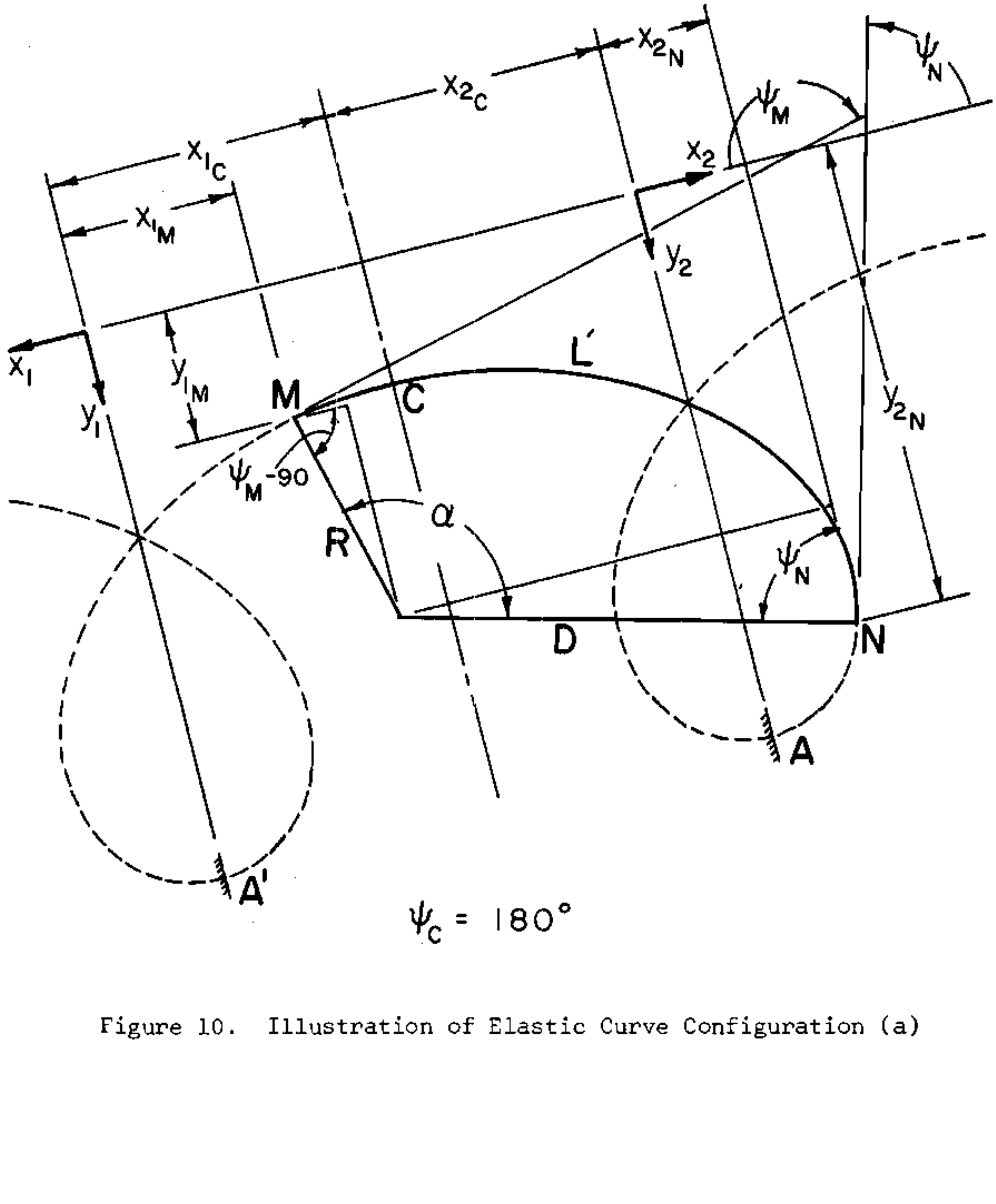


Figure 10. Illustration of Elastic Curve Configuration (a)

of p and k relate to the elastica and must be constant for all parts of the elastica. The angles the tangents from the points M and N make with the respective x-axes are ψ_M and ψ_N. For the nodal elastica the tangent to the point midway between two loops, point C, is parallel to the x-axis so $\psi_C = 180°$. Also the x-axes of all sections will lie along the same line, the line of action of the forces forming the elastica.

A set of equations from which the necessary values can be obtained is derived from the geometry of the fit as shown in Figure 10. The relation between α, ψ_M, and ψ_N is calculated by summing the values of the angles included in the angle α at the center point of the cam as follows:

$$\alpha = (180° - \psi_M) + (90° - \psi_N) + 90°$$

$$\alpha = 360° - \psi_M - \psi_N \qquad (3\text{-}7)$$

Summing the projections of the curved lengths MC and CN on the x and y axes and equating these to the sum of the projections of the lengths R and D on the same lines we obtain:

$$(x_{1_M} - x_{1_C}) + (x_{2_N} - x_{2_C}) = R\cos(\psi_M - 90) + D\sin\psi_N \qquad (3\text{-}8)$$

$$-(y_{1_M} - y_{1_C}) + (y_{2_N} - y_{2_C}) = R\sin(\psi_M - 90) + D\cos\psi_N$$

As stated previously only the orientation of the coordinate systems for different sections of the elastica differ. Equations (3-8) have been written so that they take into consideration these differences in orientation. Since the equations of defining the x and y coordinates are alike within the respective coordinate systems for the sections of the elastica, we can substitute non-subscripted x's and y's in Equations (3-8). This reduces the equations to:

$$x_M - 2x_C + x_N = R\ \sin\psi_M + D\ \sin\psi_N \tag{3-9}$$

$$-\ y_M + y_N = -R\ \cos\psi_M + D\ \cos\psi_N$$

Next we must substitute the expressions for the x and y coordinates of the elastica from Equations (3-5) into Equations (3-9). This requires the introduction of two new variables ϕ_M and ϕ_N:

$$\phi_M = \sin^{-1}(p\ \sin\frac{\psi_M}{2}) \tag{3-10}$$

$$\phi_N = \sin^{-1}(p\ \sin\frac{\psi_N}{2}) \tag{3-11}$$

$$\frac{2}{pk}\left[E(p,\frac{\psi_M}{2}) - 2E_1(p) + E(p,\frac{\psi_N}{2}) - (1 - \frac{{}^{\psi}p^2}{2})[F(p,\frac{\psi_M}{2}) - 2K(p) - F(p,\frac{\psi_N}{2})]\right]$$

$$= R\ \sin\psi_M + D\ \sin\psi_N \tag{3-12}$$

$$\frac{2}{pk}\,[\cos\phi_N - \cos\phi_M] = -R\ \cos\psi_M + D\ \cos\psi_N \tag{3-13}$$

The complete elliptic integrals result when the upper limit of the elliptic integral is equal to 90°. The expression for x_c is:

$$x_c = E\left(p,\frac{c}{2}\right) - \left(1 - \frac{p^2}{2}\right) F\left(p,\frac{c}{2}\right)$$

But ψ_c = 180°. The expression for x_c reduces to:

$$x_c = E_1(p) - \left(1 - \frac{p^2}{2}\right) K(p)$$

This accounts for the presence of the complete elliptic integrals in Equation (3-12).

Another condition which is imposed is that the cam curve and the portion of the elastica which it fits must have the same length. Therefore the length of the cam curve MN as given by the parameter L' can be equated to the length of the portion of the elastica to which it is fitted. One of the Equations (3-5) is an expression for the arc length of a basic strut measured from the ground link, point A. Therefore the length of the elastica between the points M and N is found by summing the lengths of the sections MC and NC. This length is then equated to L'. The resulting equation is:

$$L' \; (s_{1_C} - s_{1_M}) - (s_{2_C} - s_{2_M}) \qquad (3\text{-}14)$$

Like the x and y coordinate equations, the equations for the arc length within each section are the same. Therefore Equation (3-14) reduces to:

$$L' = 2s_c - s_M - s_N \qquad (3\text{-}15)$$

Substituting the expression for s from Equations (3-5), we obtain:

$$L' = \frac{p}{k}\left[2K(p) - F\left(p, \frac{\psi_M}{2}\right) - F\left(p, \frac{\psi_N}{2}\right)\right] \tag{3-16}$$

Equations (3-9), (3-10), (3-11), (3-12), (3-13), and (3-16) form a system of six equations containing six unknowns, ψ_M, ψ_N, ϕ_M, ϕ_N, p and k. Using the values for the given cam parameters R, D, α, and L', the system can be solved for the values of the unknowns which satisfy the system. A direct method of solution is impossible due to the complexity of the equations. A trial and error procedure is best used to obtain the solution as follows:

Values for the unknowns p, and ψ_M are initially assumed. The value of p must be in the range $0 < p^2 < 1$ and the value of ψ_M in the range $0 \leq \psi_M \leq 180°$. The value of ψ_N is calculated from Equation (3-7), then the values of ϕ_M and ϕ_N are calculated from Equations (3-10) and (3-11). The value of k is found from Equation (3-13). If the assumption for p is correct with respect to the value of ψ_M, then Equation (3-12) will be an equality. If the value of p is not correct, Equation (3-12) will not be satisfied, and the error, being the difference in the values of the two sides of the equation, can be calculated. Then the initially assumed value of p is adjusted until this error is reduced below a specified limit. Next, to check the assumption for ψ_M, a value of L' will be calculated from Equation (3-16) and compared with the given value of L'. If there is a difference between the two, it must be eliminated by adjusting the value of ψ_M. Of course for each adjustment of ψ_M, p must also be readjusted to maintain a value consistent with the value of ψ_M. The final values of p, ψ_M, and ψ_N must be within their respective allowable ranges. The value of p must be in the range $0 < p^2 < 1$ and all values of ψ must be in the range $0 \le \psi \le 180°$. If any of the calculated values are not in the required ranges, then the cam curve will not fit an elastica according to the assumed configuration. The values that are calculated for p, k, ψ_M, and ψ_N, and the fact that the elastica fitted is a nodal elastica, define the fit between the cam curve and the elastica for this configuration.

The configuration (a) is of special importance because it is the only configuration of the seven which can form a cam profile with a continuous acceleration over the entire profile, including the crossing at the toe, and the tangent points to the base circle. For this special case of configuration (a) an additional condition is imposed on the fitting of the cam curve to the elastica. But if an extra condition is to be added, one of the parameters must be allowed a floating value. Therefore for a cam defined by R, D, and α, there will be only one length of band which will produce a cam with continuous acceleration.

For this special case, the system of equations derived from the geometry of the cam curve and the elastica, Equations (3-7), (3-10), (3-11), (3-12), (3-13), and (3-16), is set up just as before. To these is added the condition that the radius of curvature of the band at M must equal the radius of the base circle. The radius of curvature of the band is calculated from the equation:

$$r_c(\text{radius of curvature}) = -\frac{1}{y\,k^2} \tag{3-17}$$

The procedure for the solution of the equations is the same as for the general case for configuration (a) except that the value of ψ_M is adjusted to make the radius of curvature of the band at M calculated from Equation (3-17) equal to the base circle radius R rather than making the calculated length of the band equal to a given value. Then L' is calculated from Equation (3-16).

Cam Curve Configuration (b). This configuration matches the cam curve to a portion of either the undulating or the nodal elastica. To find the values of ψ_M, ψ_N, p, and k defining the elastica and what portion of it matches the cam curve, a system of equations is derived for each type elastica. For the undulating elastica:

$$\alpha = \psi_M - \psi_N$$

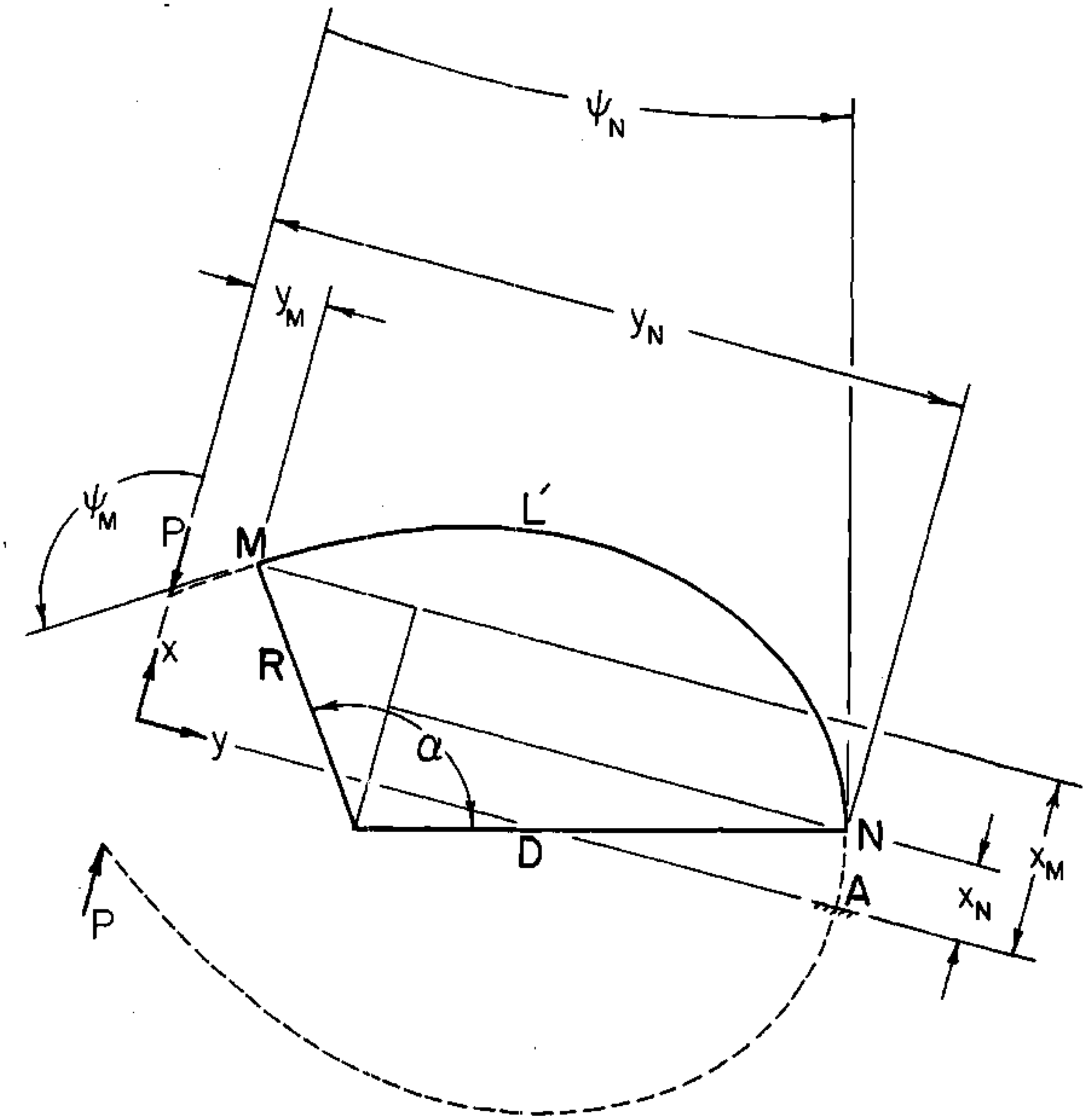
$$\phi_M = \sin^{-1}\left[\frac{\sin \psi_M/2}{p}\right]$$

$$\phi_N = \sin^{-1}\left[\frac{\sin \psi_N/2}{p}\right]$$

$$\frac{1}{k}\,[2E(p,\phi_M) - 2E(p,\phi_N) - F(p,\phi_M) + F(p,\phi_N)] \qquad (3\text{-}18)$$

$$= R \sin\phi_M - D \sin\phi_N$$

$$\frac{2p}{k}\,[\cos\phi_M - \cos\phi_N] = R \cos\phi_M - D \cos\phi_N$$



Note: The figure is drawn for undulating elastica. For the nodal elastica, the load P and the origin of the x,y coordinate system are located farther to the left along the y-axis. The general orientation of the lengths and angles of the cam curve remain the same.

Figure 11. Illustration of the Elastic Curve Configuration (b)

$$L' = \frac{1}{k}\left[F(p,\phi_M) - F(p,\phi_N)\right]$$

and for the nodal elastica:

$$\alpha = \psi_M - \psi_N$$

$$\phi_M = \sin^{-1}[p \sin \psi_M/2]$$

$$\phi_N = \sin^{-1}[p \sin \psi_N/2]$$

$$\frac{2}{pk}\left[E\left(p,\frac{\psi_M}{2}\right) - E\left(p,\frac{\psi_M}{2}\right) - \left(1 - \frac{p^2}{2}\right)\left[F\left(p,\frac{\psi_M}{2}\right) - F\left(p,\frac{\psi_N}{2}\right)\right]\right] \qquad (3\text{-}19)$$

$$= R \sin\psi_M - D \sin\psi_N$$

$$\frac{2}{pk}\left[\cos\phi_M - \cos\phi_N\right] = R \cos\psi_M - D \cos\psi_N$$

$$L' = \frac{p}{k}\left[F\left(p,\frac{\psi_M}{2}\right) - F\left(p,\frac{\psi_N}{2}\right)\right]$$

The values for p, k, ψ_M, ψ_N will all be within the allowable ranges, $0 < p^2 < 1$ and $0 \leq \psi \leq 180°$, for only one of the sets of equations since the cam curve will fit only one elastica. The values define the match between the cam curve and the elastica and the type elastica is apparent from which set of equations is used.

Cam Curve Configuration (c). The cam curve can fit either elastica for this configuration. There is no external loading on the band between the points of contact with the hub. Because of this additional condition, for given values of R, D, and α, there is only one value of L' which will form this type cam. Since there is no external load at the toe of the cam, the force P on the elastica is vertical (Figure 12). Therefore the y-axis of the elastica coincides with the centerline of the cam.

The sets of equations from which the values of p, k, ψ_M, and ψ_N can be found are: For the undulating elastica

$$\psi_M = \alpha$$

$$\psi_N = 0$$

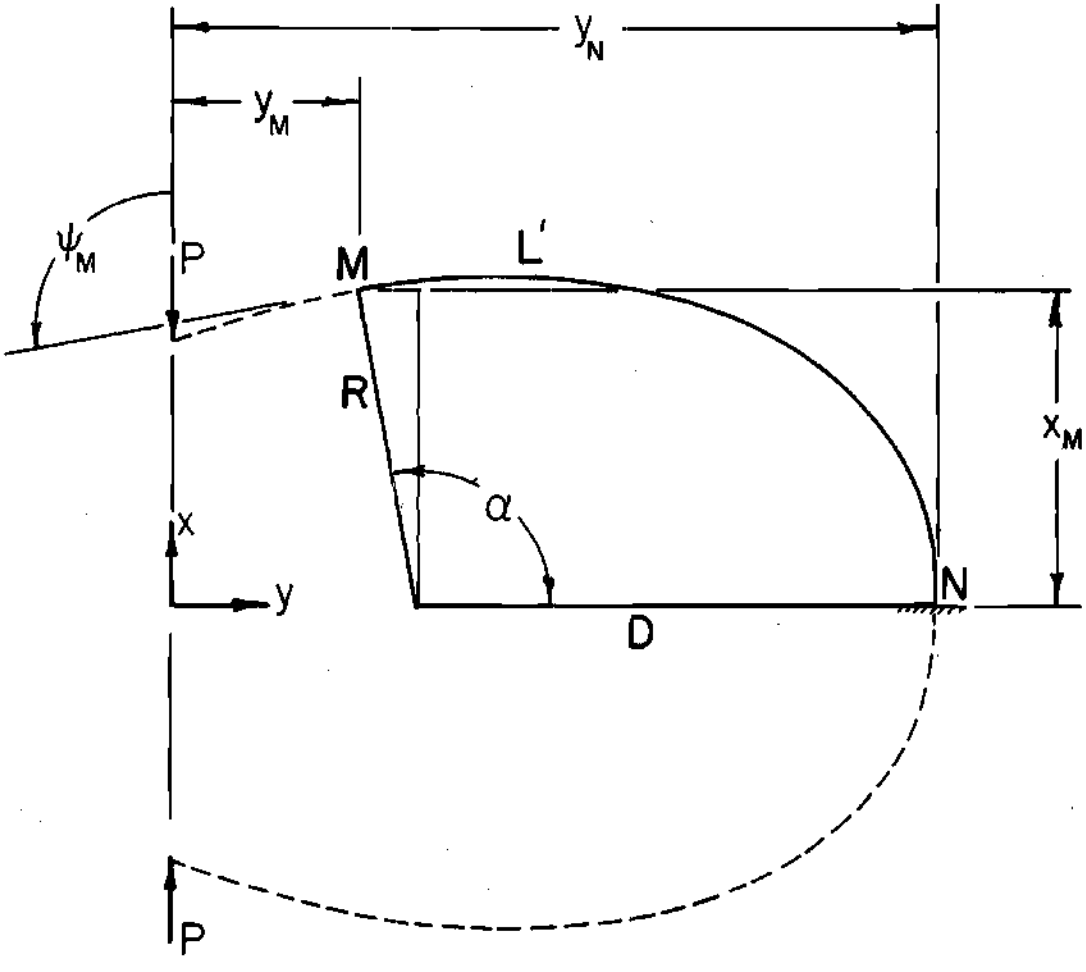
$$\phi_M = \sin^{-1}\left[\frac{\sin \alpha/2}{p}\right]$$

$$\frac{1}{k}\left[2E(p,\phi_M) - F(p,\phi_M)\right] = R \sin\alpha \tag{3-20}$$

$$\frac{2p}{k}\left[\cos\phi_M - 1\right] = D - R \cos\alpha$$

$$L' = \frac{1}{k} F(p,\phi_M)$$

and for the nodal elastica



$\psi_N = 0°$

$\psi_M = \alpha$

Figure 12. Illustration of Elastic Curve Configuration (c)

$$\psi_M = \alpha$$

$$\psi_N = 0$$

$$\phi_M = \sin^{-1}[p \sin \alpha/2]$$

$$\frac{2}{pk}\left[E\left(p,\frac{\alpha}{2}\right) - \left(1 - \frac{p^2}{2}\right) F\left(p,\frac{\alpha}{2}\right)\right] = R \sin\alpha \qquad (3\text{-}21)$$

$$\frac{2}{pk}\left[\cos\phi_M - 1\right] = D - R \cos\alpha$$

$$L' = \frac{p}{k} F\left(p,\frac{\alpha}{2}\right)$$

The set of equations whose solution for p, k, ψ_M, and ψ_N which is within the allowable range define the match between the cam curve and the elastica.

Cam Curve Configuration (d). The cam curve can match a portion of either elastica. The sets of equations from which the values of p, k, ψ_M, and ψ_N can be found are:

For the undulating elastica

$$\alpha = \psi_M + \psi_N$$

$$\phi_M = \sin^{-1}\left[\frac{\sin \psi_M/2}{p}\right]$$

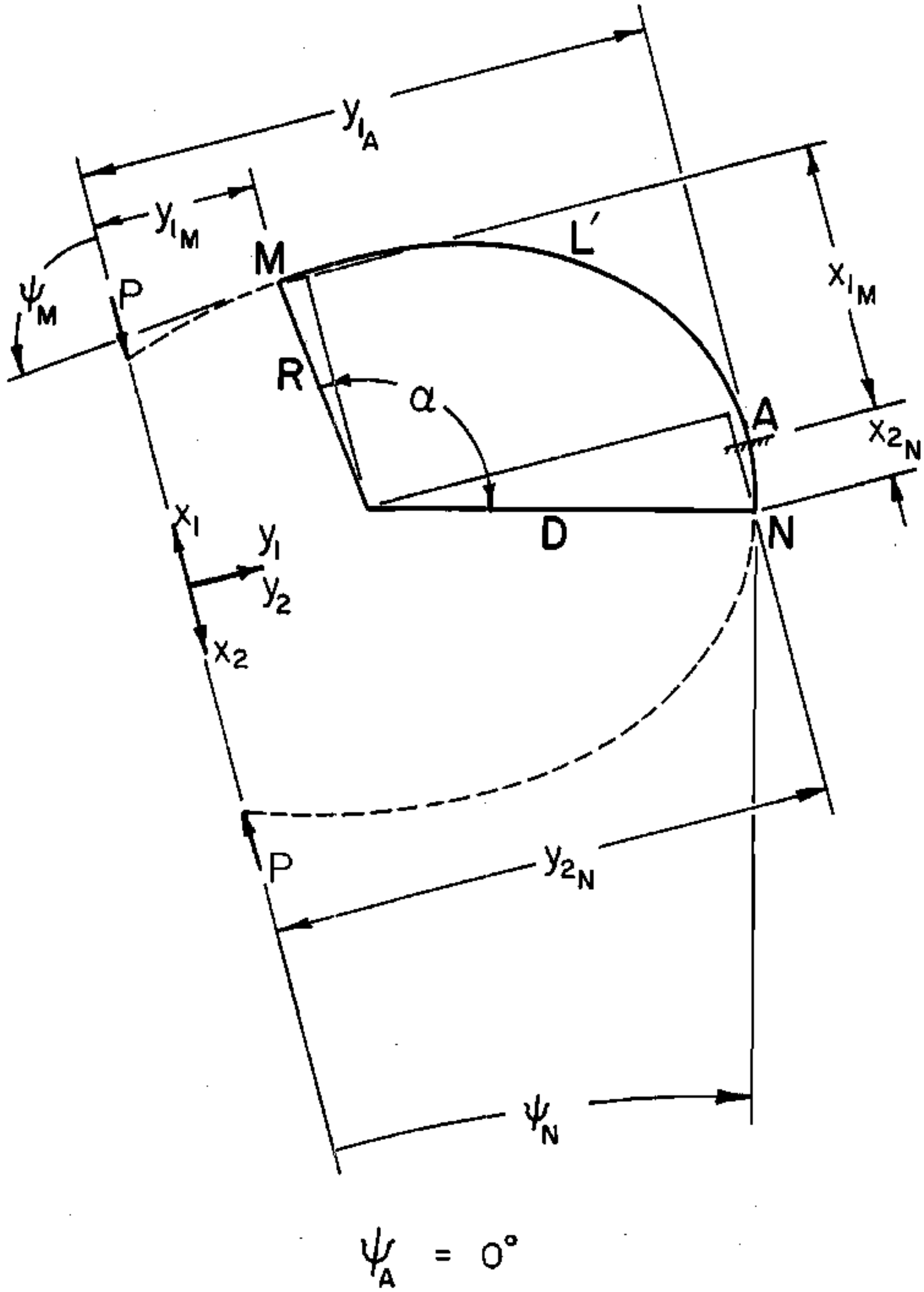


Figure 13. Illustration of Elastic Curve Configuration (d)

$$\phi_N = \sin^{-1}\left[\frac{\sin \psi_N/2}{p}\right]$$

$$\frac{1}{k}\,[2E(p,\phi_M) + 2E(p,\phi_N) - F(p,\phi_M) - F(p,\phi_N)] \tag{3-22}$$

$$= R \sin\psi_M + D \sin\psi_N$$

$$\frac{2p}{k}\,[\cos\phi_M - \cos\phi_N] = R \cos\psi_M - D \cos\psi_N$$

$$L' = \frac{1}{k}\,[F(p,\phi_M) + F(p,\phi_N)]$$

and for the nodal elastica

$$\alpha = \psi_M + \psi_N$$

$$\phi_M = \sin^{-1}\left[p \sin \frac{\psi_M}{2}\right]$$

$$\phi_N = \sin^{-1}\left[p \sin \frac{\psi_N}{2}\right]$$

$$\frac{2}{pk}\left[E\left(p,\frac{\psi_M}{2}\right) + E\left(p,\frac{\psi_N}{2}\right) - \left(1 - \frac{p^2}{2}\right)\left[F\left(p,\frac{\psi_M}{2}\right) + F\left(p,\frac{\psi_M}{2}\right)\right]\right] \tag{3-23}$$

$$= R \sin\psi_M + D \sin\psi_N$$

$$\frac{2}{pk}\,[\cos\phi_M - \cos\phi_N] = R \cos\psi_M - D \cos\psi_N$$

$$L' = \frac{p}{k}\left[F\left(p,\frac{\psi_M}{2}\right) + F\left(p,\frac{\psi_N}{2}\right)\right]$$

The set of equations whose solution for p, k, ψ_M, and ψ_N which is within the allowable range define the match between the cam curve and the elastica.

Cam Curve Configuration (e). The cam curve will only match a portion of the undulating elastica since the elastic curve contains a point of contraflexure. The set of equations from which the parameters p, k, ψ_M, and ψ_N defining the fit between the cam curve and the elastica are:

$$\alpha = \psi_M - \psi_N$$

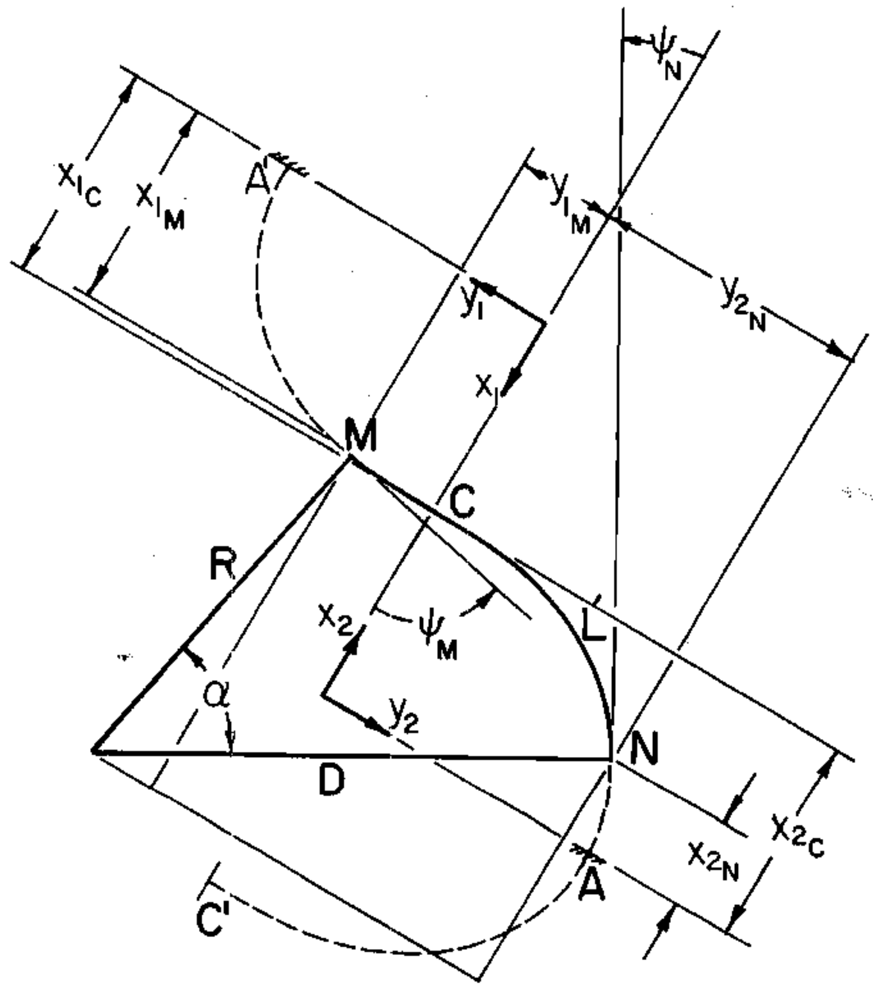
$$\phi_M = \sin^{-1}\left[\frac{\sin \psi_M/2}{p}\right]$$

$$\phi_N = \sin^{-1}\left[\frac{\sin \psi_N/2}{p}\right]$$

$$\frac{1}{k}\,[4\,E_1(p) - 2E(p,\phi_M) - 2E(p,\phi_N) \tag{3-24}$$

$$- 2K(p) + F(p,\phi_M) + F(p,\phi_N)]$$

$$= R\,\sin\psi_M - D\,\sin\psi_N$$



$\phi_c = 90°$

Figure 14. Illustration of Elastic Curve Configuration (e)

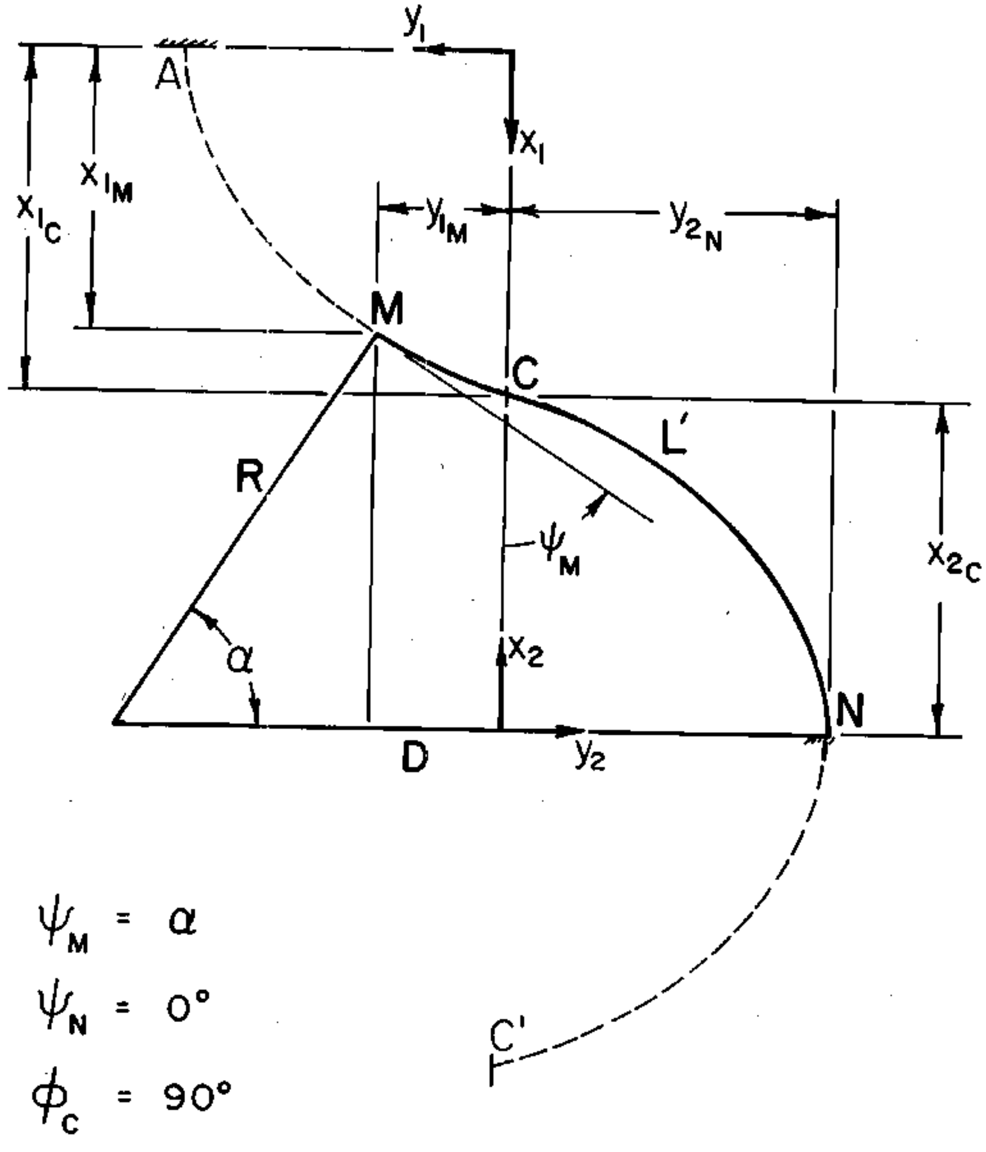


Figure 15. Illustration of Elastic Curve Configuration (f)

$$\frac{2p}{k}[\cos\phi_M + \cos\phi_N] = -R\cos\psi_M + D\cos\psi_N$$

$$L' = \frac{1}{k}[2K(p) - F(p,\phi_M) - F(p,\phi_N)]$$

Cam Curve Configuration (f). The cam curve in this form has no external load at the toe of the cam and a point of contraflexure. Therefore, it can be fitted only to the undulating elastica with the y_2-axis of the elastica coinciding with the cam centerline. The set of equations from which the parameters p, k, ψ_M, and ψ_N defining the fit between the cam curve and the elastica are:

$$\psi_M = \alpha$$

$$\psi_N = 0$$

$$\phi_M = \sin^{-1}\left[\frac{\sin\alpha/2}{p}\right]$$

$$\phi_c = 0$$

$$\frac{1}{k}[4E_1(p) - 2E(p,\phi_M) - 2K(p) + F(p,\phi_M)] \tag{3-25}$$

$$= R\sin\alpha$$

$$\frac{2p}{k}[1 - \cos\phi_M] = D - R\cos\alpha$$

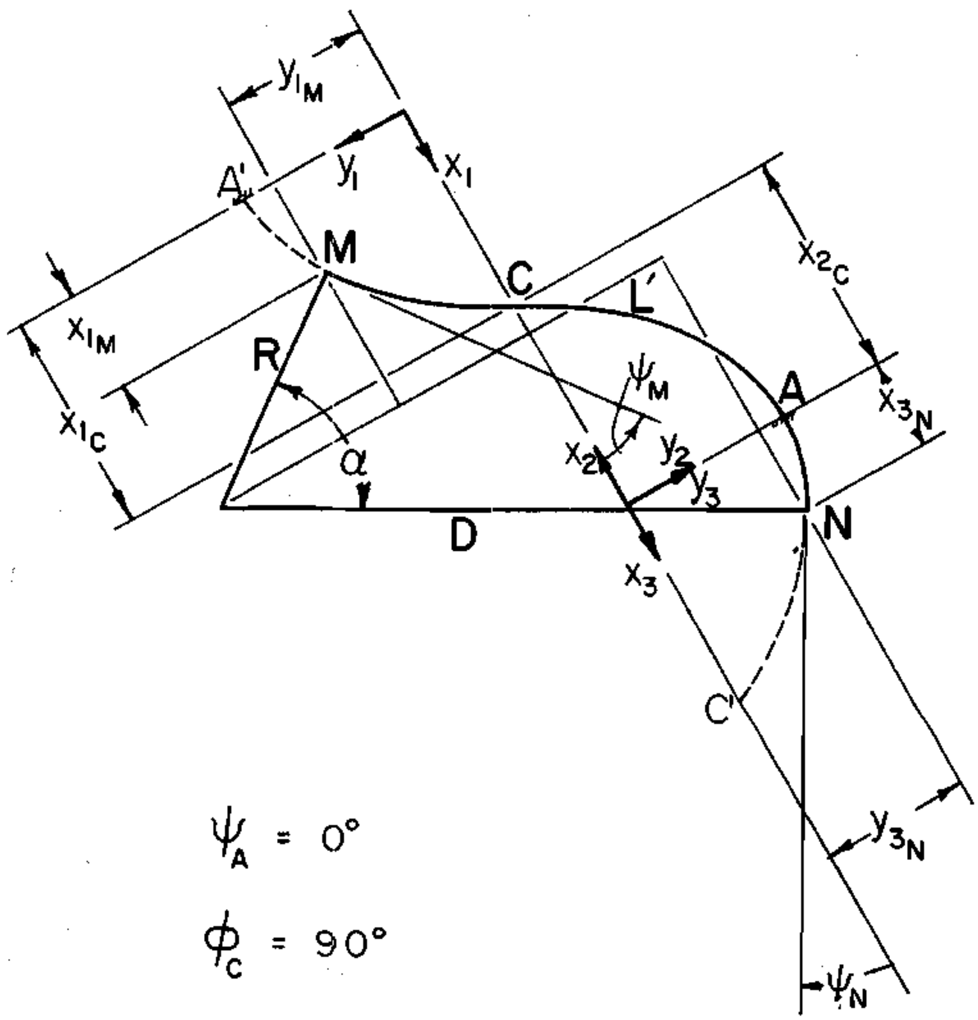


Figure 16. Illustration of Elastic Curve Configuration (g)

$$L' = \frac{1}{k}\left[2K(p) - F(p,\phi_M)\right]$$

Cam Curve Configuration (g). This cam curve will only match a portion of the undulating elastica since the curve contains a point of contraflexure. The set of equations from which the parameters p, k, ψ_M, and ψ_N defining the fit between the cam curve and the elastica are:

$$\alpha = \psi_M + \psi_N$$

$$\phi_M = \sin^{-1}\left[\frac{\sin \psi_M/2}{p}\right]$$

$$\phi_N = \sin^{-1}\left[\frac{\sin \psi_N/2}{p}\right]$$

$$\frac{1}{k}[4E_1(p) - E(p,\phi_M) + E(p,\phi_N) \qquad (3\text{-}26)$$

$$- 2K(p) + F(p,\phi_M) - F(p,\phi_N)$$

$$= R \cos\psi_M + D \cos\psi_N$$

$$\frac{2p}{k}[\cos\phi_M + \cos\phi_N] = -R \sin\psi_M + D \sin\psi_N$$

$$L' = \frac{1}{k}[2K(p) - F(p,\phi_M) + F(p,\phi_N)]$$

Selecting the Correct Fit Between the Cam Curve and an Elastica

Given that a cam curve matches an elastica according to one of the configurations (a) through (g), methods have just been presented which show how to select the elastica, specified by the type, p, and k, and what portion of that elastica, specified by the angles ψ_M and ψ_N, which will fit the cam curve. The next step is to be able to select the correct match between a cam curve to an elastica when the cam curve is specified in terms of the cam parameters R, D, α, and L. This is not a simple task because the cam parameters do not give a direct indication of which configuration the cam curve will fit. Therefore, a particular configuration must be assumed and then proven to be either correct or incorrect. The proof is to solve the equations associated with the configurations (a) through (g) for values of p, k, ψ_M, and ψ_N. If the values calculated for all of these unknowns lie within the acceptable limits, $0 < p^2 < 1$ and $0 \leq \psi \leq 180°$, then the cam curve fits an elastica according to the specified form. If any one of the variables is outside its acceptable range, then the cam curve does not match the chosen configuration.

The next question is how can one assume a form which will be the correct one and if this form which is selected is incorrect, which one should be tried next? For the special case of cams with continuous acceleration there is no problem for they must always fit configuration (a). This case cannot exist when the cam curve must have a reverse curvature in order to form the cam curve. This will occur when a tangent to the base circle from the point M intersects the cam centerline inside the point N or only a short distance outside this point. The reverse curvature in the band for such cases is necessary if the cam curve is to conform to the necessary end conditions. As would be expected cam curves with a reverse curvature only occur when α is less than 90 degrees.

The other special case is the cam with a free elastic band. This case will fit either configuration (c) or (e). Which of these a particular cam curve fits depends upon whether a reverse curvature is necessary to form the cam.

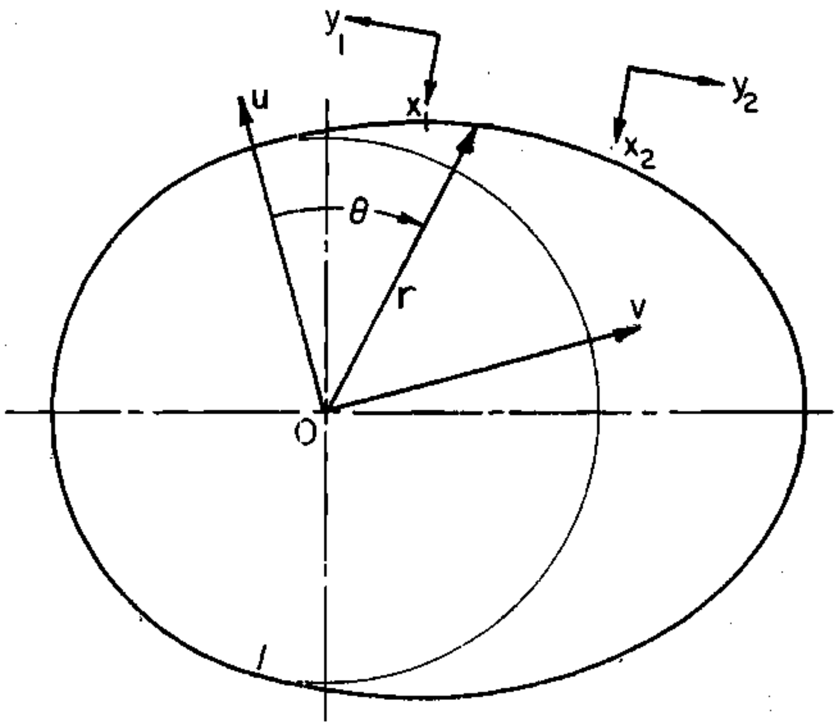
For fixed values of R, D, and α, the length L of the band required to form the special case of a cam with continuous acceleration, is minimum. The length of band necessary to form the special cases with a free elastic band is an intermediate length. The values of the length of band required for these two special cases are given in Tables 3 and 5 for a cam of unit base circle radius and various rise. By comparing the length of band of a given cam with the values in these tables for a comparable cam, a prediction can be made of which configuration the cam curve will fit. For example, if the length of band for a cam is greater than the length of band which would form a cam with a free elastic band, then the cam would fit either configuration (d) or (g) depending on whether it required a reverse curvature. If a cam is to be formed with a length of band which is longer than the length forming the continuous acceleration cam, yet shorter than the length forming the cam with a free elastic band, it must fit one of three configurations: (a), (b), or (e). Neither configuration (a) nor (b) includes a reverse curvature. As would be expected from an inspection of Figure 8, the closer the length of the band to the minimum length, the more likely the cam curve will fit configuration (a); and the closer to the free elastic band, the more likely it will fit configuration (b). Of course if a reverse curvature is required, the cam curve will fit configuration (e). When there is no value of length given in Table 2 for the cam with continuous acceleration, this means the minimum length band forms a cam curve with a reverse curvature. Hence all cams with the same R, D, and α, but longer bands will also have reverse curvature forms.

Using this approach, the correct match can be selected for most cam curves. If an incorrect selection occurs it is usually because the necessity for a reverse curvature in the cam curve was not foreseen. The worst possible situation would occur if two incorrect selections were made before the correct one was found.

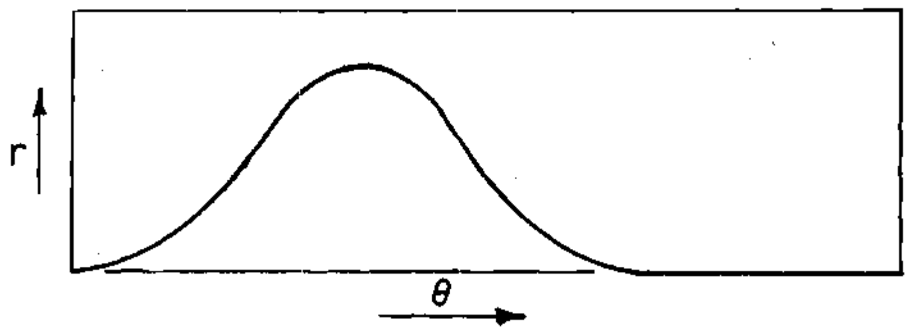
The Equations of the Cam Profile in the r,Θ Coordinate System

The methods for defining the fit between a cam curve and an elastica have been established. The results from these are values for p, k, ψ_M, and ψ_N, the type of elastica fitted, undulating or nodal, and which of the seven configurations, (a) through (g), the cam curve fits. Using this information the equations describing the cam profile can be found. The first step in forming these equations is to set up the coordinate systems that will be used.

Frisch-Fay described the shape of the elasticas in terms of a series of orthogonal coordinates systems x,y, one for each basic strut section from which the elastica was formed. (Figures 2 and 5). However, the shape of the cam would best be described using polar coordinates centered in the axis of the hub of the cam. In order to correlate



Example for Configuration (a)



Displacement Diagram

Figure 17. Coordinate Systems Associated with the Analysis of the Elastic Band Cam

these, it is best to transform the Equations (3-4) and (3-5), the equations describing the elastica in the x,y system, by two stages, first from the x,y systems of Frisch-Fay to a skew orthogonal system u,v with its origin at point 0 of the cam (Figure 17). Secondly, the shape is transformed to a polar coordinate system r,Θ whose origin is also at the point 0 of the cam.

The transformation from the x,y system to the u,v system is effected by the following equations:

$$u = C_1[(y_M - C_2 y)\cos\psi + (x_M - C_4 - C_3 x)\sin\psi_M] + R \qquad (3\text{-}27)$$

$$v = C_1[(y_M - C_2 y)\sin\psi_M - (x_M - C_4 - C_3 x)\cos\psi_M]$$

The values of the constants C_1, C_2, C_3, and C_4 are given in Table 1 for the different configurations. There are different values of the constants within the configurations which encompass more than one basic strut section of an elastica. The value of ψ_M is known from the fit of the cam curve to the elastica, and x_M, y_M and x_c are the x,y coordinates of the points M and C, respectively, calculated from either Equations (3-4) or (3-5). Then the u,v system is transformed to the r,Θ system by the equations:

$$r = \sqrt{u^2 + v^2} \qquad (3\text{-}28)$$

$$\Theta = \tan^{-1}[v/u]$$

Table 1. Values of Constants Used in Equations (3-27) for the Transformation from the x,y Coordinate System to the u,v System

Configuration	x,y System Number	C_1	C_2	C_3	C_4
(a)	1	-1	+1	+1	0
	2	-1	+1	-1	$2x_c$
(b)	-	-1	+1	+1	0
(c)	-	-1	+1	+1	0
(d)	1	-1	+1	+1	0
	2	-1	+1	-1	0
(e)	1	+1	+1	+1	0
	2	+1	-1	-1	$2x_c$
(f)	1	+1	+1	+1	0
	2	+1	-1	-1	$2x_c$
(g)	1	+1	+1	+1	0
	2	+1	-1	-1	$2x_c$
	3	+1	-1	+1	$2x_c$

Equations (3-28) are actually the equations of the cam profile in parametric form in terms of the parameters u and v. In turn, Equations (3-27) define u and v in terms of the parameters x and y. Finally, x and y are defined by either Equations (3-4) or (3-5), depending upon whether they are for an undulating or a nodal elastica, in terms of a single parameter ψ. The value of ψ varies from ψ_M to ψ_N to describe the rise portion of the cam. The path of variation for ψ differs with the configuration of the fit between the cam curve and the elastica. These paths are given in Table 2.

Table 2. Progression of ψ from ψ_M to ψ_N for the Different Cam Curve Configurations

Configuration	
(a)	ψ_M increases to ψ_C decreases to ψ_N
(b)	ψ_M decreases to ψ_N
(c)	ψ_M decreases to ψ_N
(d)	ψ_M decreases to ψ_A increases to ψ_N
(e)	ψ_M increases to ψ_C decreases to ψ_N
(f)	ψ_M increases to ψ_C decreases to ψ_N
(g)	ψ_M increases to ψ_C decreases to ψ_A increases to ψ_N

$\psi_A = 0$ $\psi_c = 180°$

When the equations for the curve in the x,y coordinate system, Equations (3-4) or (3-5), are substituted into Equations (3-27) and the resulting equations are then substituted into Equations (3-28), a complicated definition of the cam profile in the r,Θ system including many elliptic integrals results. Rather than work directly from these equations to form the displacement curve of the cam, a series x,y coordinate points along the curve is calculated. The coordinate of each point are transformed into the u,v system, and then to the r,Θ system. The cam curve or displacement diagram in the r,Θ system is described by this series of coordinate points. Also finding the derivatives of r with respect to Θ directly from the equations is not the most practical way. An alternate method is to calculate the velocity and acceleration curves for the cam by the numerical differentiation of the displacement curve.

Equations (3-27) are only of use when working with the rise portion of the cam curve from M to N (Figure 7). The symmetry of the cam is utilized to produce the return portion of the cam curve in the r,Θ system from the rise portion.

Results from the Analysis of Elastic Band Cams

The mathematical representations of the cam profiles can be used to tabulate a set of data describing the characteristics of the cam profiles. But the motion of the cam follower is the important motion rather than the cam contour and therefore the follower path must be presented in order that the data be useful for design work. There are four basic follower systems considered: The radial translating roller, the radial translating flat-faced, the oscillating roller, and the oscillating flat-faced. For each of these systems there are an infinite number of combinations of dimensions. Therefore for any given cam profile, there are an infinite number of variations to be investigated resulting from follower types. In order to present the data in the most efficient manner, first the data pertaining to the cam acceleration curves are presented. Then, a second set of data is presented showing the relationships between the cam acceleration curves and the follower acceleration curves for various follower systems. This second set of data is used in conjunction with the data on cam profiles to predict the follower motion characteristics from the cam motion characteristics.

The data necessary to produce the charts of cam and follower characteristics was obtained from an analysis of a large number of elastic band cams. The necessity of using trial-and-error methods to establish the fit between the cam curve and an elastica, the calculation and transformation of a series of precision points describing the cam curve, the numerical differentiation to find the velocity and acceleration curves, and the large number of cams to be analyzed, were all factors necessitating the use of a computer to calculate the required data. A separate program was used for each of the seven configurations, (a) through (g) and one for the special case for a cam with continuous acceleration. These could have all been included in one program, but it would have been too large and complicated to be an improvement. Each program was set up so that for a given set of cam parameters, it would:

1. Calculate the values of p, k, ψ_M, and ψ_N, provided they were all within the correct ranges, for a fit with the appropriate type of elastica.

2. Calculate a series of coordinate points describing the rise portion of the cam curve, and transform them into the r,Θ coordinate

system forming the displacement diagram for the cam.

3. For a particular type of follower system, calculate the path of the follower from the displacement diagram of the cam.

4. Using numerical differentiation, calculate the velocity and acceleration of the cam and the acceleration of the follower path.

5. Print out the cam parameters, the variables defining the fit between the cam curve and the elastica, and the displacement and acceleration curves, in tabular form, for the cam and follower.

These computer programs are included in Appendix B.

Characteristics of Cam Profiles

For work up to this point an elastic band cam has been described by four parameters: the angle of rise (α), the distance from the center to the toe (D), the length of the band (L), and the radius of the base circle (R). Now, so that the tabulated data will be applicable to a cam of any size, a new set of dimensionless parameters is defined by dividing all parameters of length by the base circle radius. The resulting set of parameters is: The angle of rise (α), the "rise ratio" $\left(\frac{D-R}{R}\right)$, and the "length ratio" (L/R). This also decreases the number of parameters and makes the data tabulation easier.

For any given angle of rise and rise ratio there is a family of elastic band cams produced as the length ratio is varied. An acceleration curve can be plotted versus the camshaft displacement for each cam in this family. Figure 18 is an example. The acceleration ratio, the ratio of the actual acceleration divided by the angular velocity squared to the radius of the base circle $\left(\frac{a/\omega^2}{R}\right)$, rather than the actual value

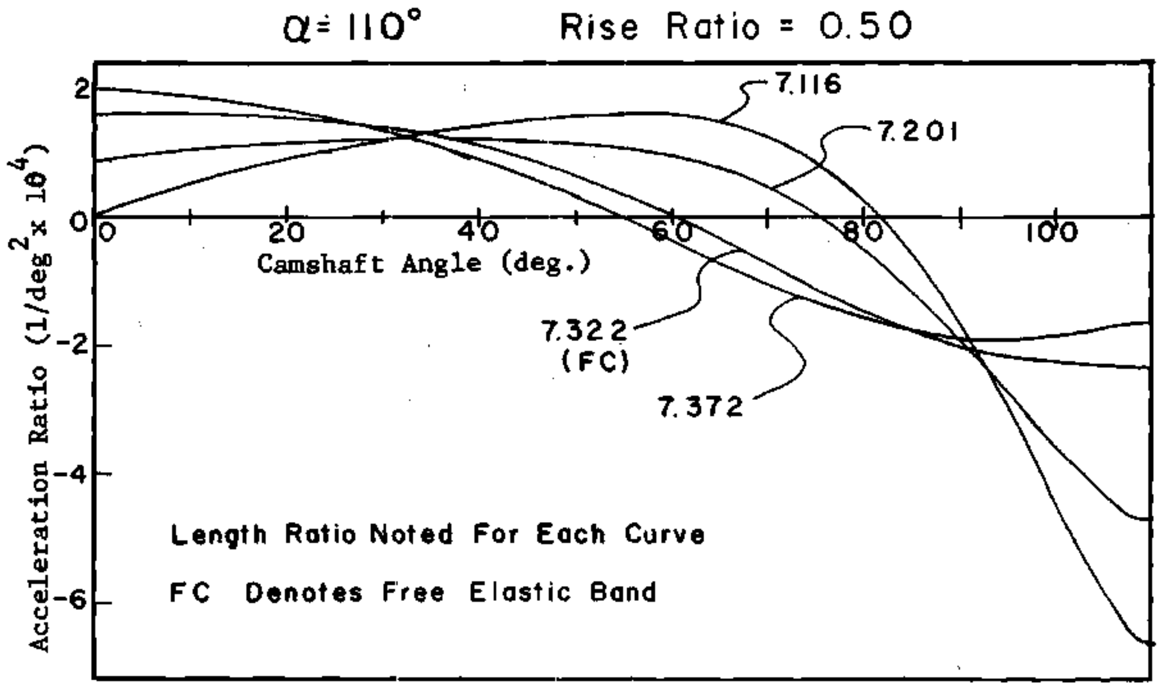


Figure 18. Example Data Chart Showing Family of Acceleration Curves

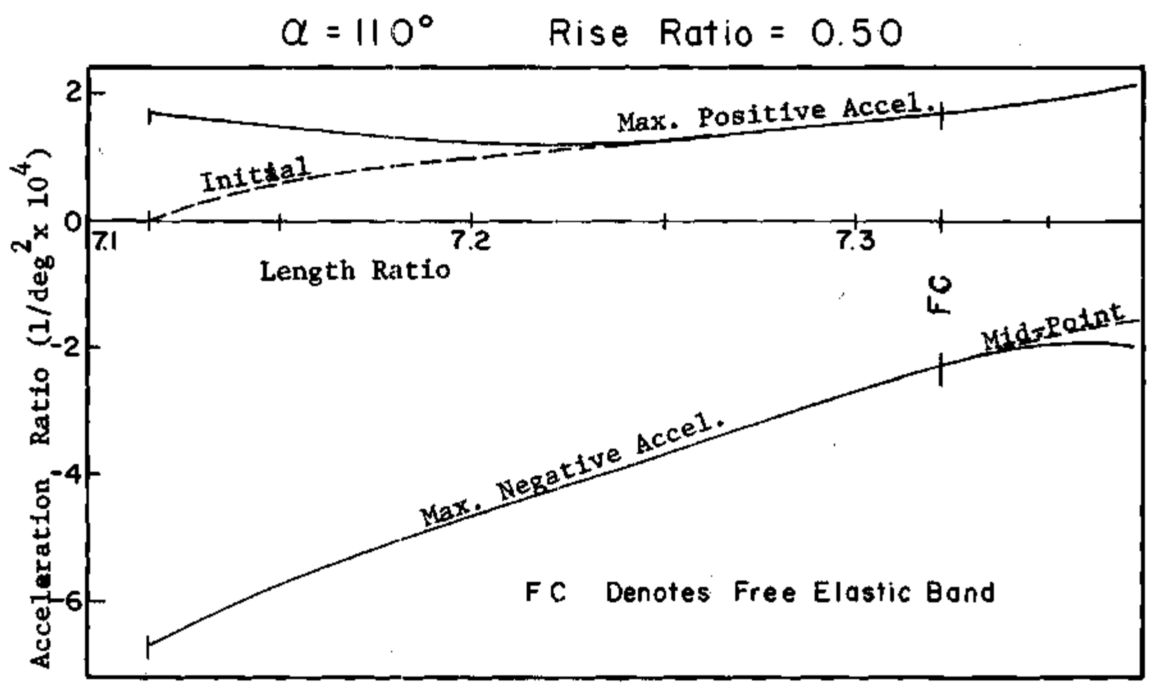


Figure 19. Example Data Chart Giving Acceleration Characteristics

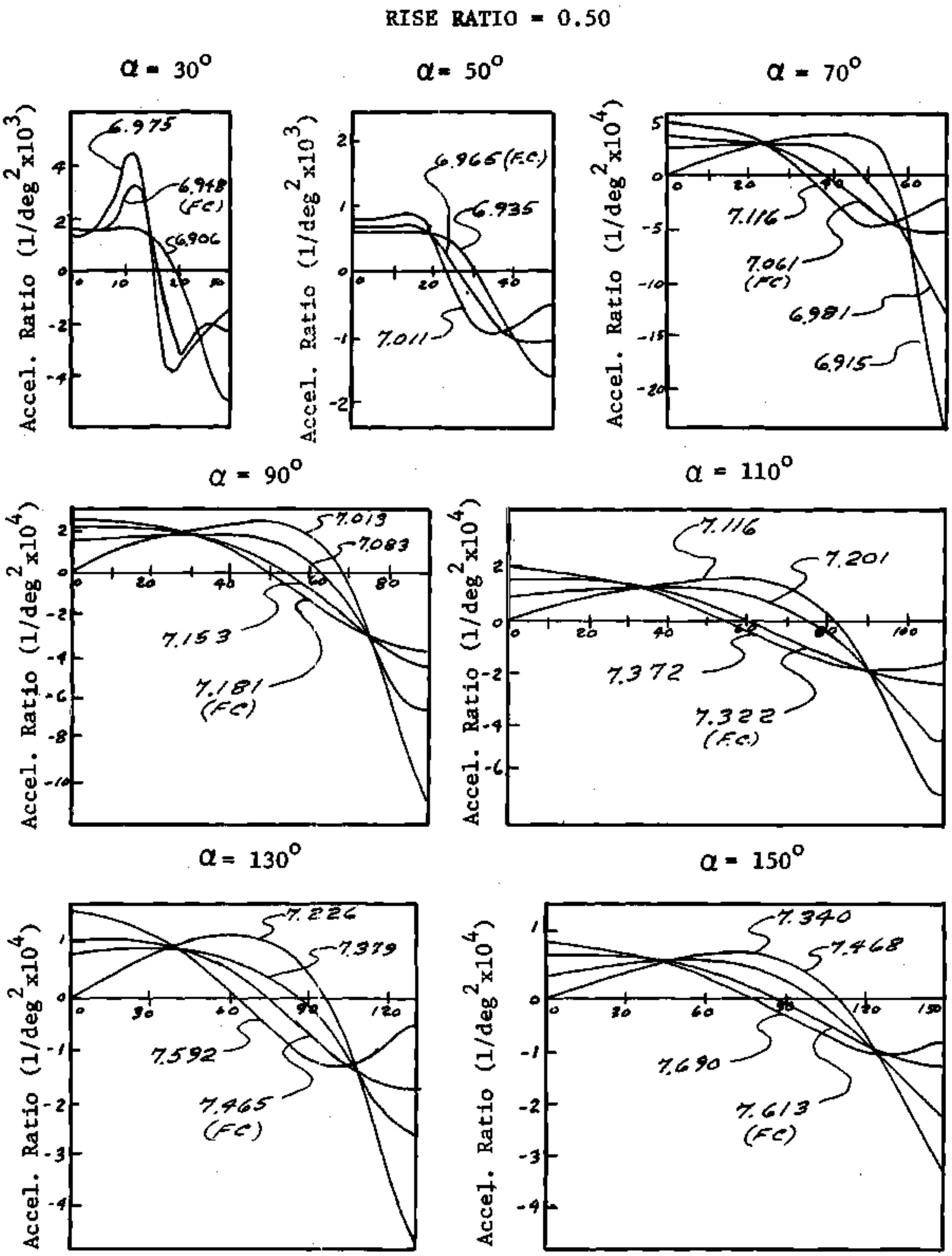


Figure 20. Families of Acceleration Curves for Elastic Band Cams

for the acceleration, is used to plot these curves in order that it be the output of a cam defined by dimensionless parameters.

The general shape of all of the acceleration curves for the basic elastic band cam is the same. The curves in Figure 18 are typical. All of the elastic band cams are symmetric so the acceleration curves are symmetric about the point of maximum rise. For this reason, only the rise portion of the curve is presented, the return portion being the mirror image.

In any one family of cams having the same angle of rise and rise ratio (Figure 18), there is at most one cam which has continuous acceleration. Unfortunately this cam also has the highest maximum negative acceleration ratio. The other cams in the family have positive values of acceleration at the initial point, but have lesser values of the maximum acceleration.

In Figure 20 further examples of the acceleration curves of the elastic band cams are shown. All charts are for a rise ratio equal to 0.50 and angle of rise as noted above each chart. The length ratio for each curve is noted on a leader. The length ratio for cams having a free elastic band are noted by (FC) near the value of the length ratio.

For some low angles of rise there is no cam in the family which has a continuous acceleration. This occurs when the cam curve has a reverse curvature and does not fit configuration (a) of Figure 8, but fits configuration (e) of Figure 9, for a band of minimum length.

The bulk of the information for the evaluation of the cam profiles is presented on charts having curves showing the maximum positive acceleration ratio, the maximum negative acceleration ratio, the acceleration ratio at the initial point of rise when it differs from the maximum positive acceleration ratio, and the acceleration ratio at the point of maximum rise when it differs from the maximum negative acceleration ratio, all plotted versus the length ratio. Figure 19 illustrates. This type of chart was plotted rather than the acceleration curves because it is easier to read and use. The information included is that which is most often taken from the acceleration curves. For each of four values of rise ratio the charts are included for seven values of the angle of rise (Figures 21, 22, 23, and 24). The maximum acceleration, etc., for a set of parameters for which a chart is not plotted can be predicted by interpolating between the given charts.

A special case of the elastic band cam is the configuration where the band is not acted on by any external force between the points of contact with the hub. A cam built to fit this type of configuration is easy to design and produce, and has the minimum values of maximum accelerations, but its acceleration will have a finite jump at the point of initial rise. This would exclude its use for high speed operation where the dynamics of the system are critical. Its simplicity makes it useful for low speed applications. Charts showing the acceleration characteristics of this type cam are plotted versus the rise ratio in Figure 25.

In Tables 3 and 5 values of the length ratio necessary to form special cases of cams with continuous acceleration and with a free elastic band are presented. Table 4 gives values of the modulus p and of k for the elastic curve of cams with continuous acceleration. These values define the exact elastica which the particular cam will fit.

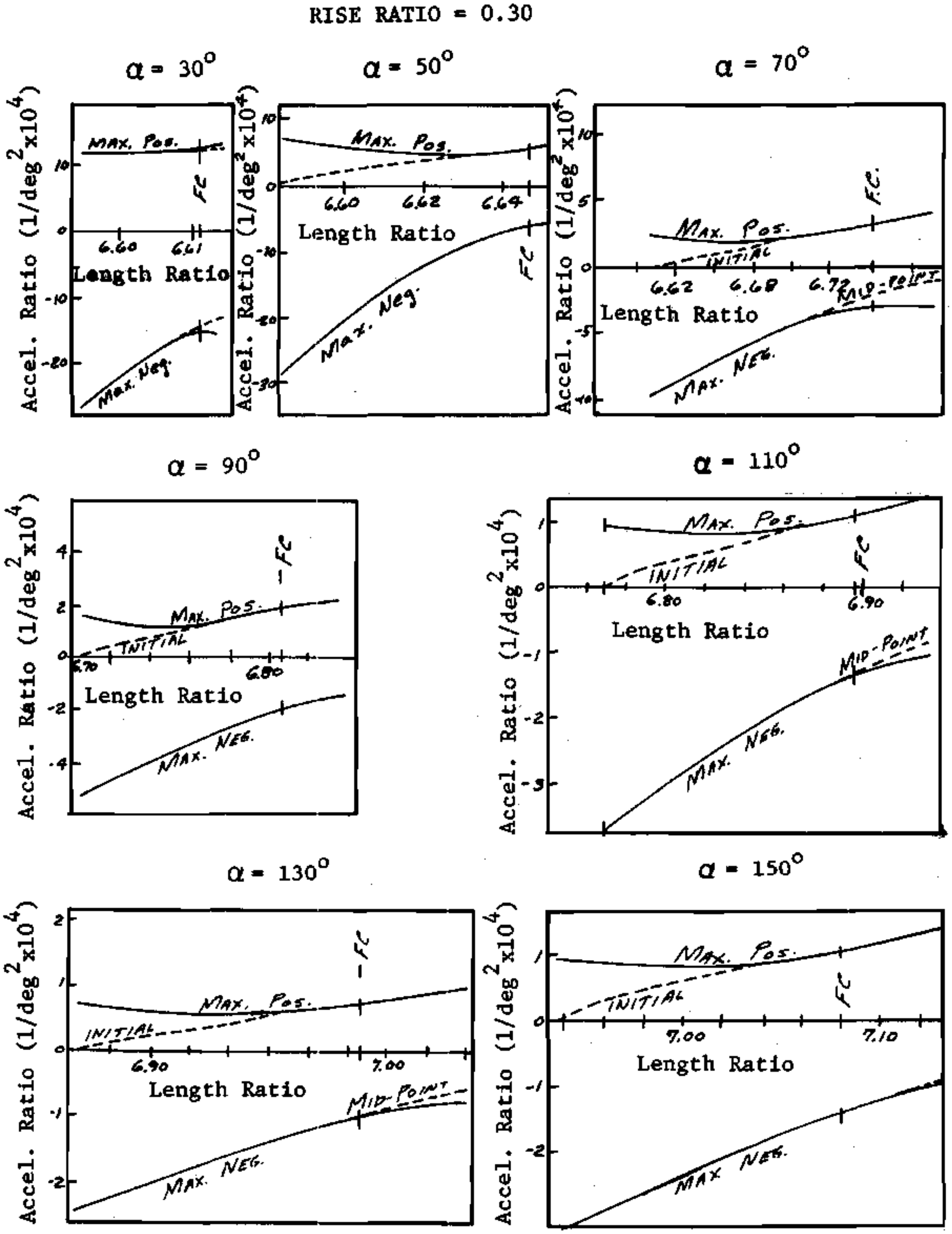


Figure 21. Acceleration Characteristic Charts for Elastic Band Cams with Rise Ratio Equal 0.30

RISE RATIO = 0.50

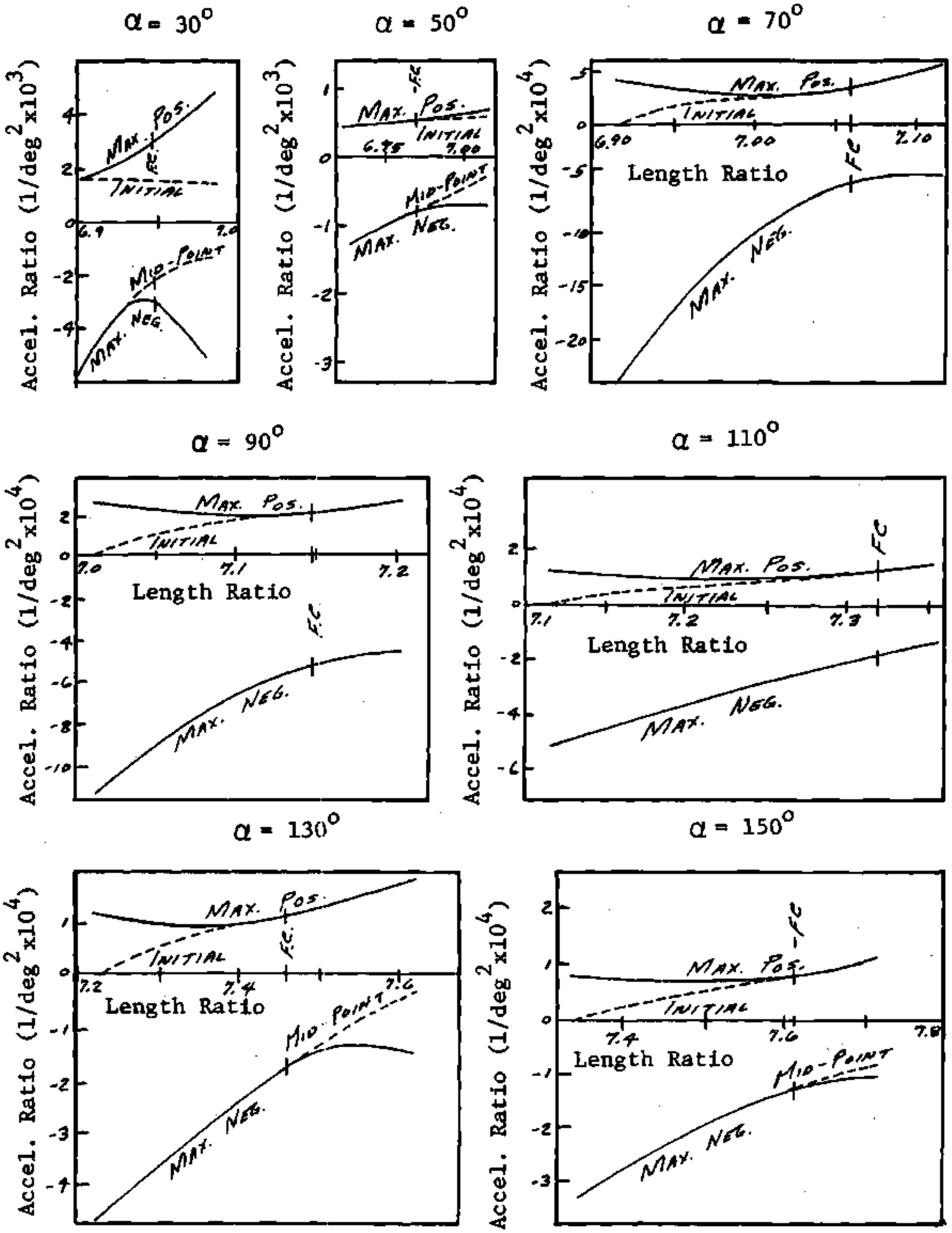


Figure 22. Acceleration Characteristic Charts for Elastic Band Cams with Rise Ratio Equal 0.50

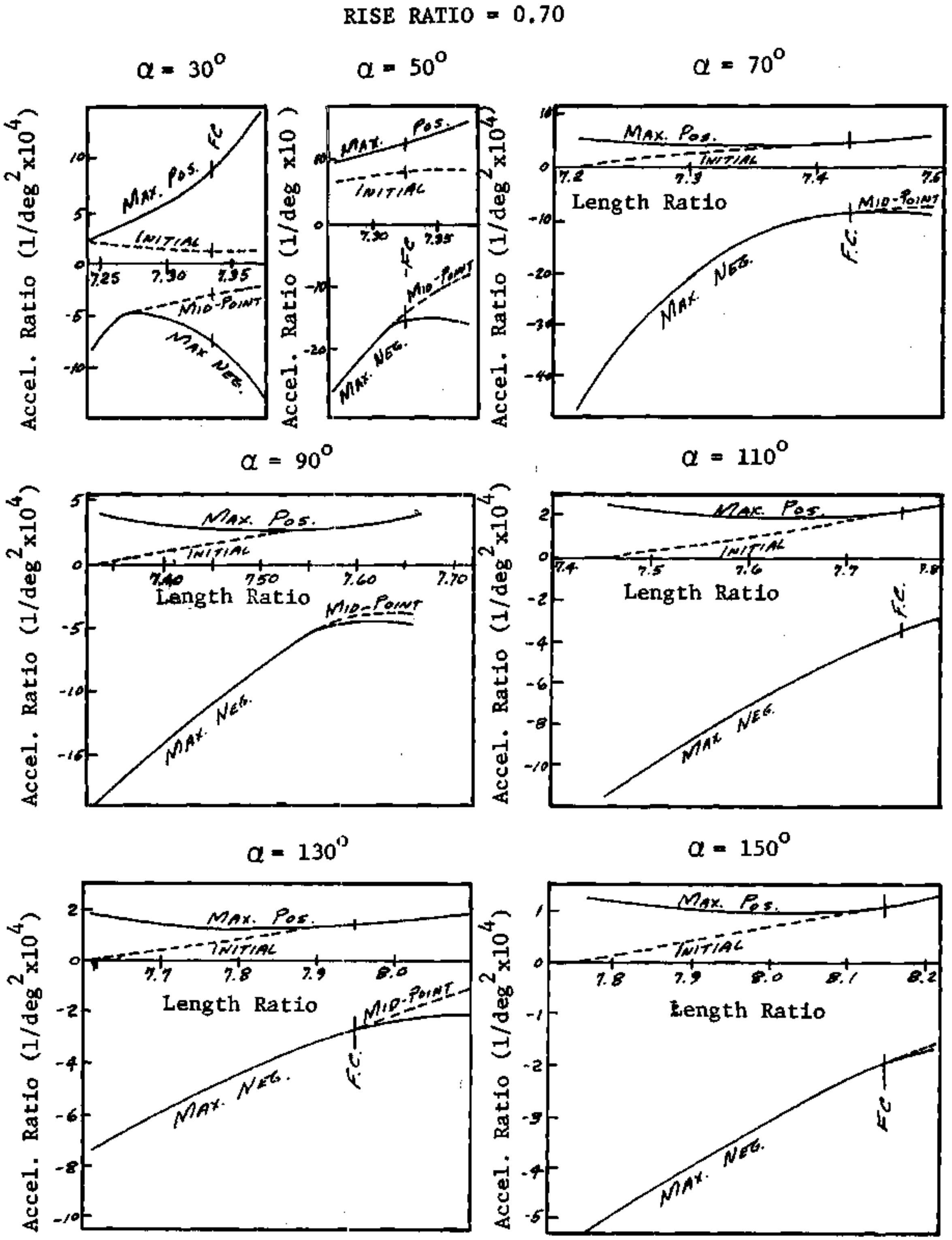


Figure 23. Acceleration Characteristic Charts for Elastic Band Cams with Rise Ratio Equal 0.70

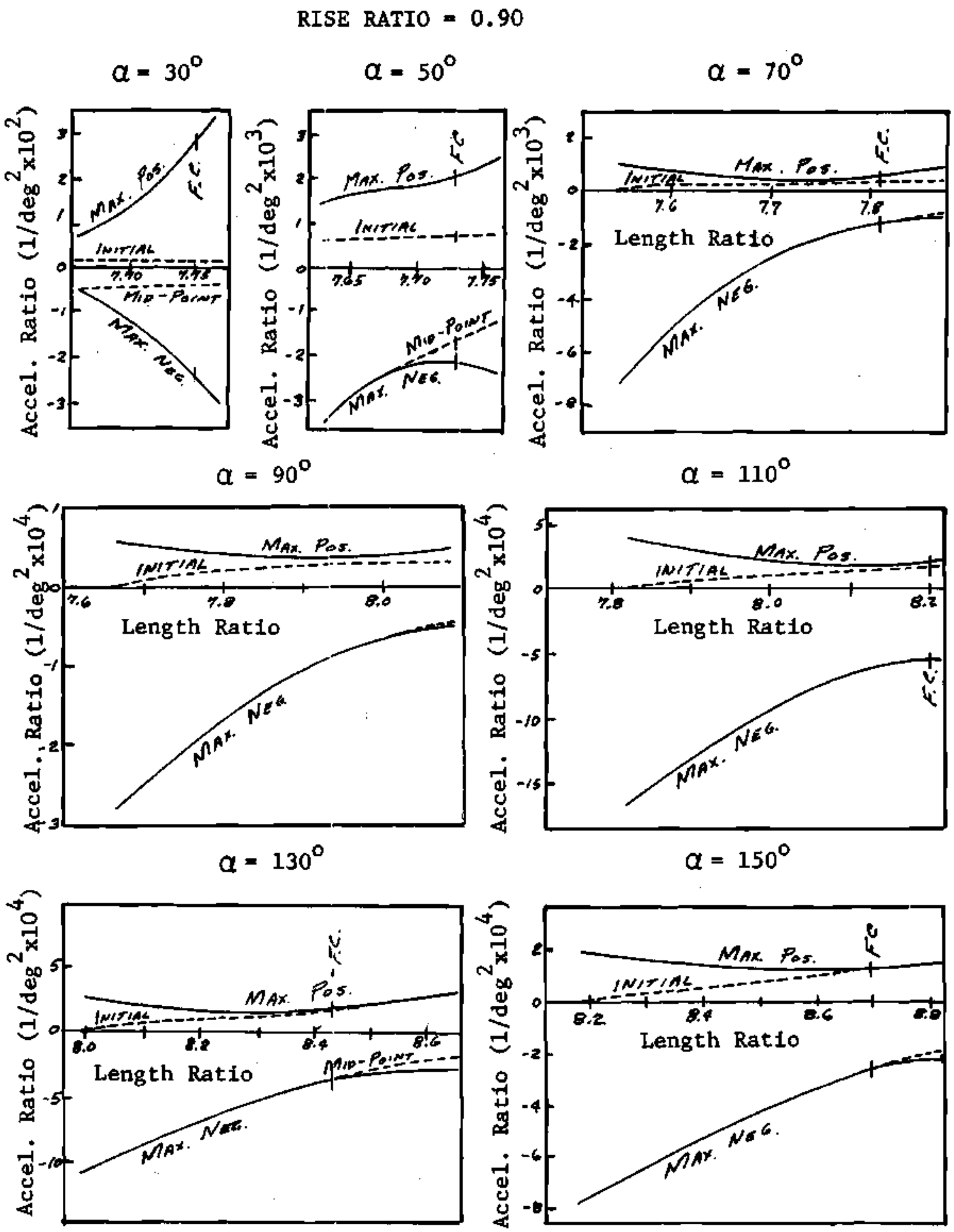


Figure 24. Acceleration Characteristic Charts for Elastic Band Cams with Rise Ratio Equal 0.90

CAM WITH FREE ELASTIC BAND

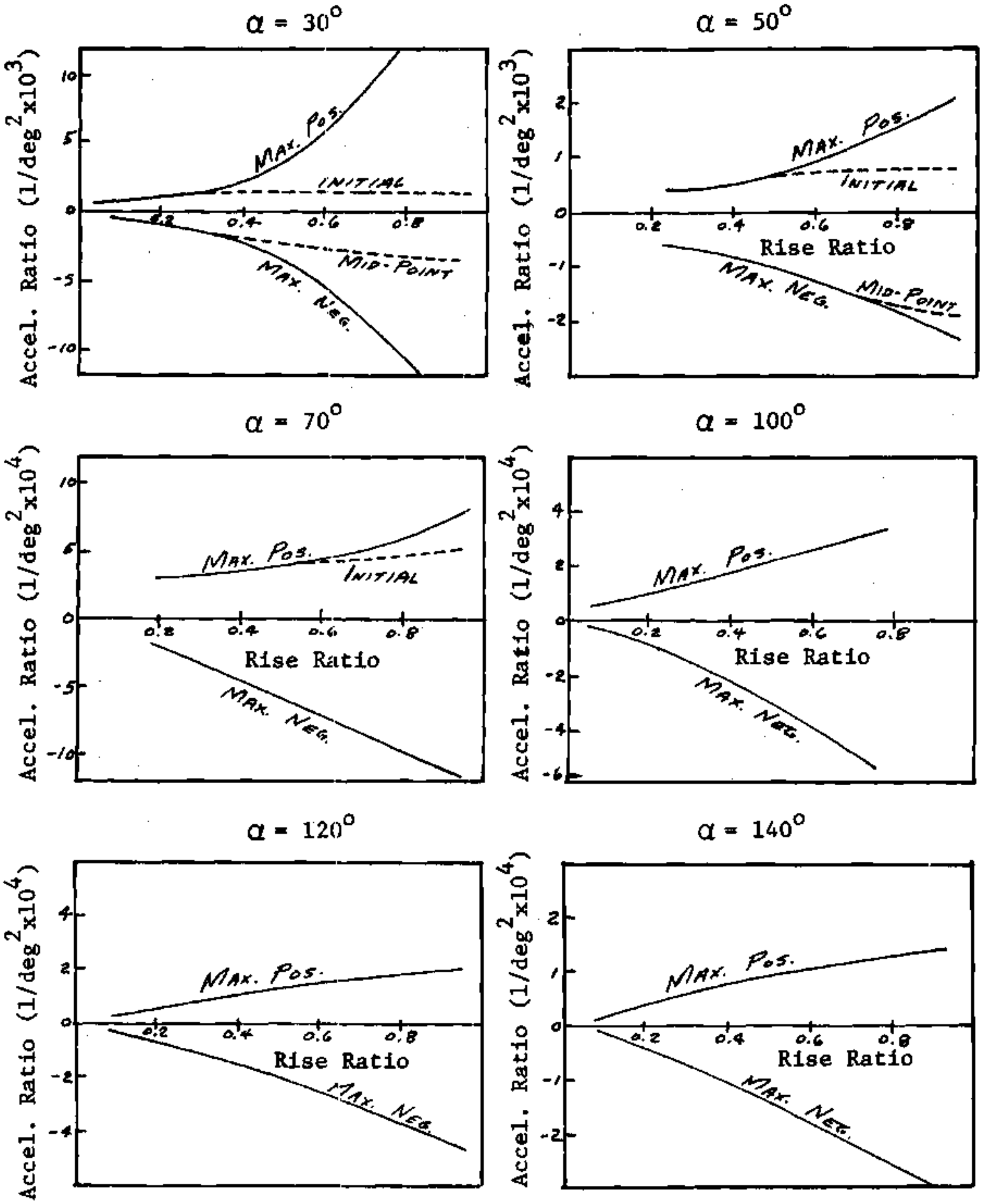


Figure 25. Acceleration Characteristic Charts for Cams with a Free Elastic Band

Table 3. Values of Length Ratio for Elastic Band Cams Which Have Continuous Acceleration

Angle of Rise	RISE RATIO					
	0.10	0.30	0.50	0.70	0.90	1.10
50°	6.3648	6.5823				
70°	6.3888	6.6359	6.9184			
90°	6.4147	6.7003	7.0092	7.3376	7.6739	
110°	6.4414	6.7709	7.1134	7.4662	7.8260	8.1918
130°	6.4688	6.8452	7.2265	7.6116	7.9989	8.3882
150°	6.4936	6.9214	7.3446	7.7657	8.1850	8.6019

A simplified procedure can then be used to find the location on this elastica that the cam curve fits. Setting the radius of curvature at M equal to the base circle radius, and substituting this and the value of k into Equation (3-17), y_M can be calculated. Then substituting y_M and p into Equation (3-5) for the y coordinate, ϕ_M is found. From the definition of ϕ in terms of p and ψ in Equations (3-5), ψ_M is found. The value of ψ_N is obtained from Equation (3-7). The values of ψ_M and ψ_N locate the cam curve on the nodal elastica defined by p and k.

Table 4. Values of the Modulus p and k for the Elastic Curve of Cams Which Have Continuous Acceleration

VALUES OF p

Angle of Rise	Rise Ratio 0.10	0.30	0.50	0.70	0.90	1.10
50°	0.99773	0.99999				
70°	0.97959	0.99847	0.99988			
90°	0.92941	0.98704	0.99626	0.99881	0.99962	
110°	0.84102	0.95561	0.98097	0.99057	0.99498	0.99723
130°	0.72641	0.89838	0.94686	0.96838	0.97981	0.98656
150°	0.60734	0.81812	0.89172	0.92854	0.94991	0.96364

VALUES OF k

Angle of Rise	0.10	0.30	0.50	0.70	0.90	1.10
50°	7.77524	20.16671				
70°	3.60693	6.48551	8.93807			
90°	2.09985	3.50674	4.38589	5.05648	5.62010	
110°	1.37106	2.22989	2.71530	3.04302	3.28284	3.46789
130°	0.96121	1.54751	1.86627	2.07419	2.21909	2.32538
150°	0.70794	1.13678	1.36838	1.51895	1.62282	1.69927

Table 5. Values of Length Ratio for Elastic Band Cams Which Have a Free Elastic Band

Angle of Rise	RISE RATIO				
	0.10	0.30	0.50	0.70	0.90
30°	6.3586	6.6092	6.9482	7.3370	7.7546
50°	6.3884	6.6476	6.9652	7.3218	7.7054
70°	6.4182	6.7314	7.0607	7.4262	7.8114
90°	6.4509	6.8070	7.1807	7.5830	8.0059
110°	6.4839	6.8962	7.3218	7.7572	8.2004
130°	6.5157	6.9870	7.4646	7.9472	8.4338
150°	6.5497	7.0808	7.6128	8.1481	8.6850

<u>Characteristics of the Follower Paths</u>

The next step in evaluating the cams is to determine the shape of the follower paths. Four types of follower systems are considered as mentioned previously. A set of data charts is given which is used in connection with the cam profile data to predict the path of the follower on an elastic band cam. Each chart is for a constant value of angle of rise and length ratio and includes acceleration curves for the cam contour and for examples of the four types of followers. Figure 26 illustrates. The procedures which are used to calculate the follower paths from the cam profiles are given in Appendix C, and the computer programs to effect these are given in Appendix B.

The dimensions of a follower system will affect its acceleration.

It is evident that it would be impossible to show the acceleration curves for follower systems with all possible combinations of dimensions. Hence, it is necessary to select a few systems for presentation.

The set of charts in Figures 27 through 32 show the acceleration curves of follower systems operating on elastic band cams. The dimensions of all follower systems used on these charts are presented in dimensionless form by dividing them by the base circle radius.

Oscillating follower systems will be defined by the vertical and horizontal distances between the cam shaft center and the follower pivot, the arm length, and the roller radius or distance from follower arm centerline to the follower face, whichever is applicable. For the flat-faced oscillating follower, the acceleration curve for only one system is given on each chart. In this system the horizontal distance ratio from the camshaft to the follower is eight and the vertical distance ratio is two. There is no measurable arm length, but the distance ratio from the arm centerline to the face is 0.75. The acceleration curves for more than one oscillating roller follower system are given on charts (Figures 26-32). Each system has a different arm length. All other dimensions are constants: the roller radius ratio equals 0.75 the vertical distance between the camshaft and the follower pivot equals 2.00, and the horizontal distance from the camshaft to the follower pivot equals the arm length. The arm length ratio is indicated on the charts with a number enclosed in a square. The characteristics of follower systems with dimensions different from those given can be predicted from the available information. For example, the acceleration for an oscillating follower system is approximately inversely

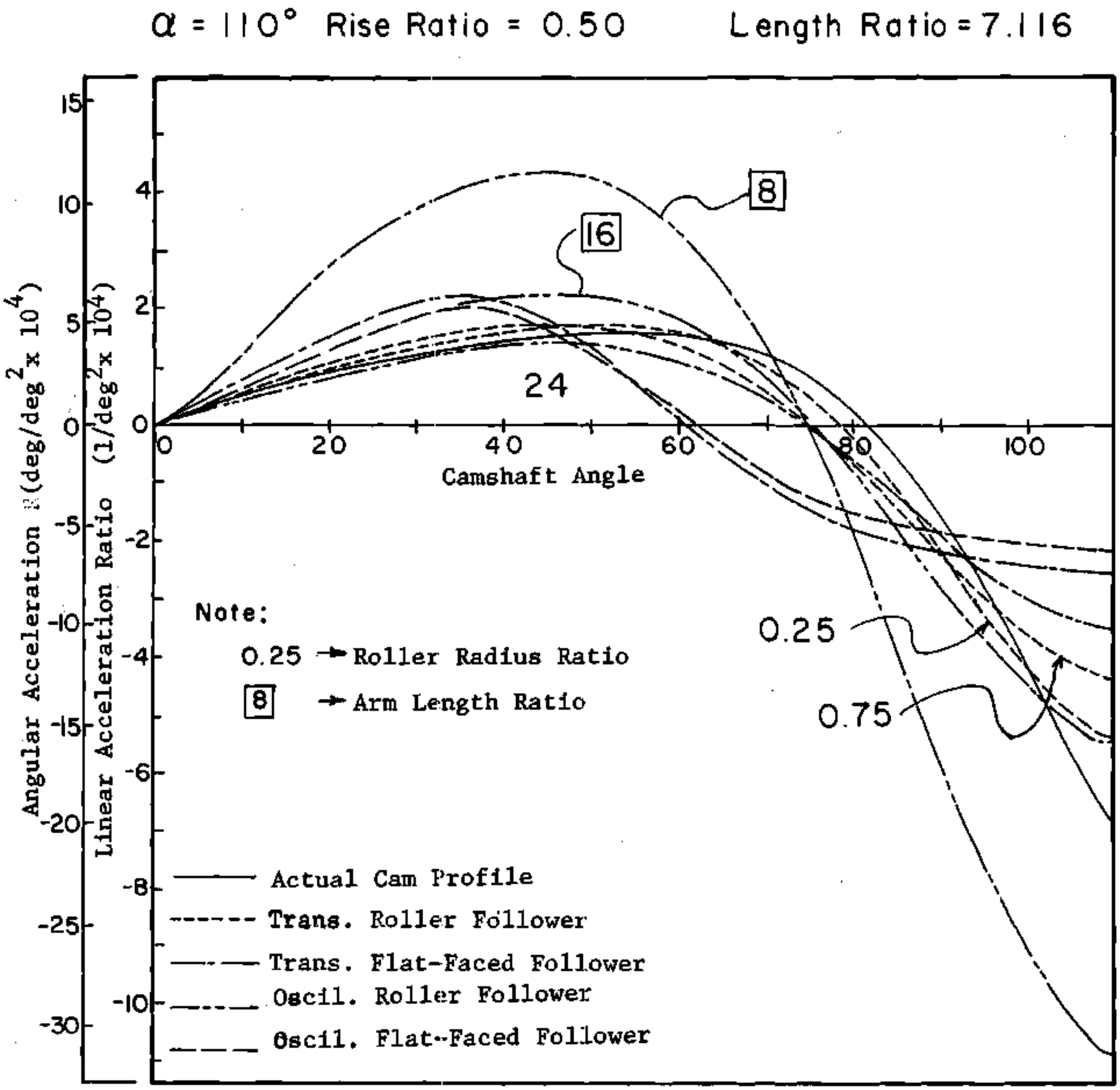


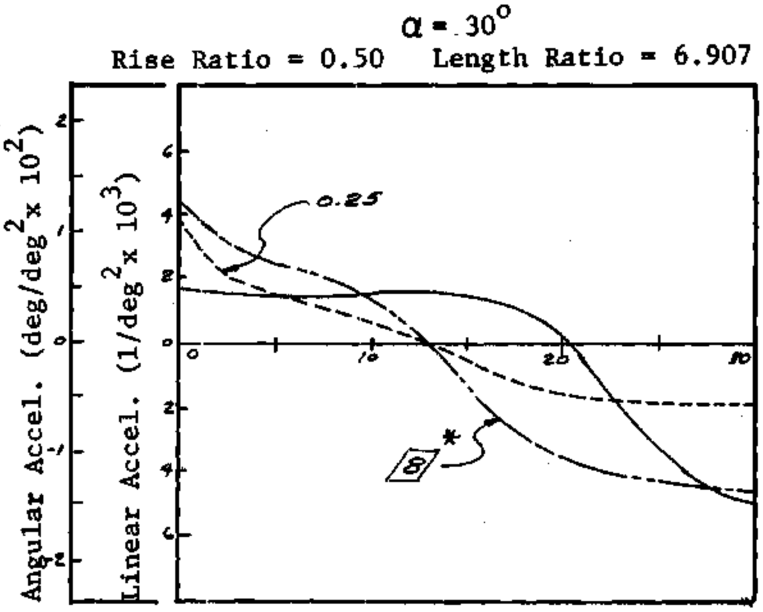
Figure 26. Example Chart for the Comparison of Cam Profile Acceleration with Follower Accelerations

proportional to the arm length.

The path of the translating flat-faced follower is unaffected by dimensional changes so only one acceleration curve for it is considered. For translating roller followers the path changes for different roller sizes and offsets. The paths for a translating roller follower with two different roller radius ratios and zero offset are shown on the charts. As roller radius decreases, the follower path tends to become the same as the shape of the cam profile. If the roller size increases, the shape of the path tends to the path of the translating flat-faced follower. Therefore, the paths of other translating roller followers can be predicted. The value for the roller radius ratio is noted on a leader to the acceleration curve.

The type of follower system represented by a curve on these charts is determined from the legend of lines given on Figure 26. On some charts there are no acceleration curves for flat-faced follower systems because these cams have a negative curvature. Therefore, they can never be used with flat-faced followers.

The changes from the actual cam acceleration to the accelerations of the followers can be observed on the follower output charts. These charts are used along with the cam characteristic charts to predict the follower motion on a given cam. The follower acceleration for cams not included in the set of follower output charts can be found by interpolating between the given charts. Therefore, given the characteristics of any elastic band cam, the effect of a follower on it can be predicted. An example design problem presented in Chapter V illustrates the prediction of follower acceleration from the cam characteristics.



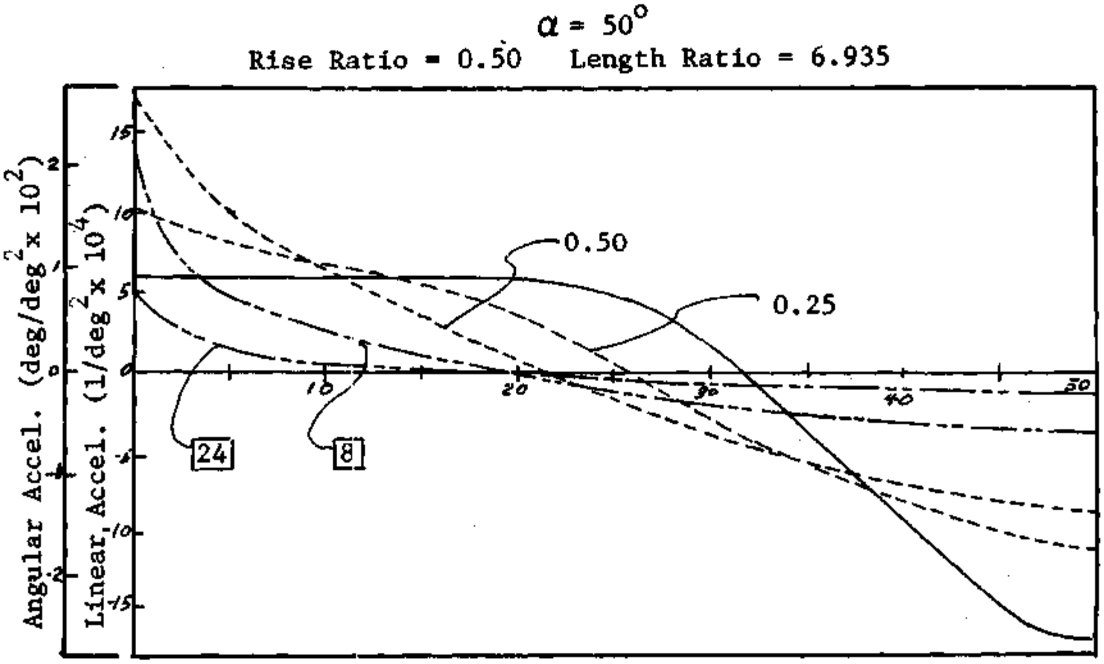


Figure 27. Follower Output Charts

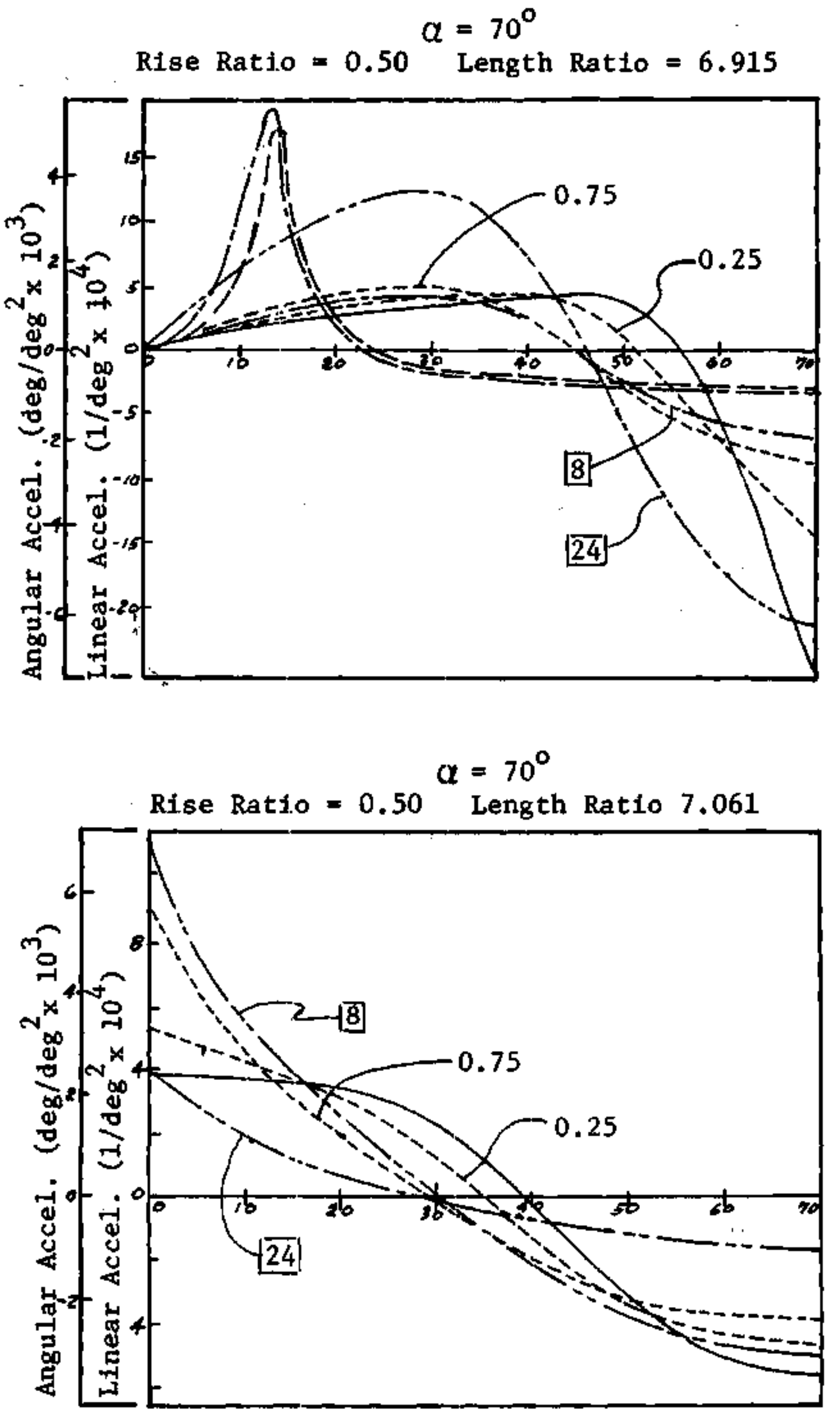


Figure 28. Follower Output Charts

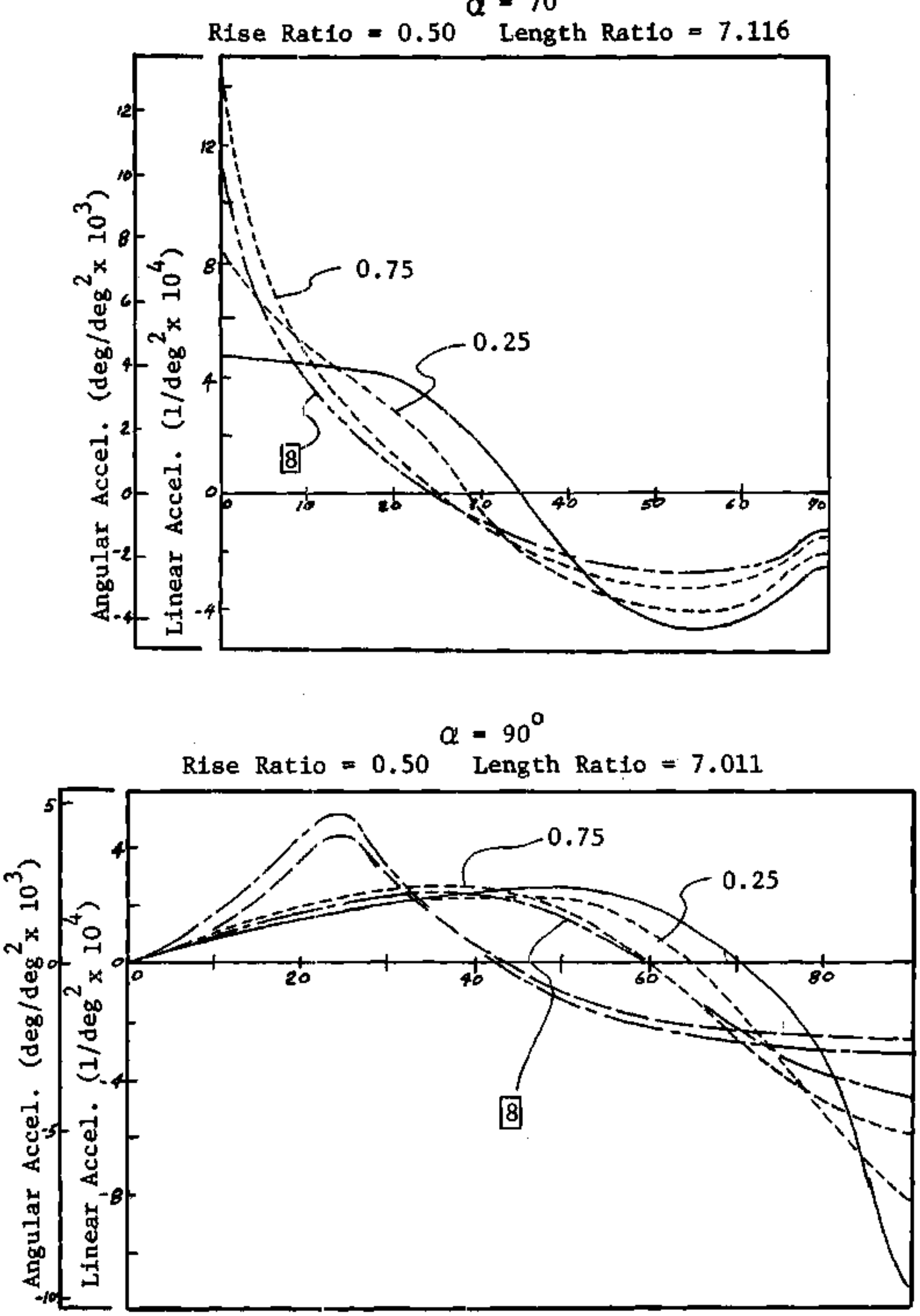
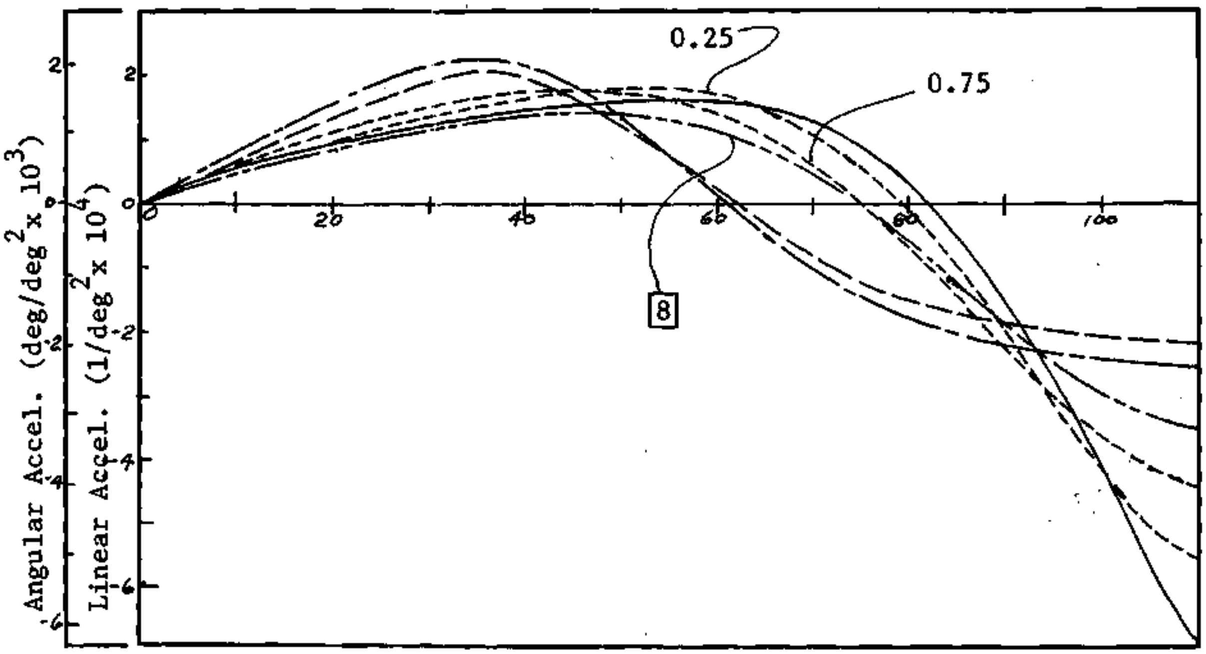


Figure 29. Follower Output Charts

$\alpha = 110^{\circ}$
Rise Ratio = 0.50   Length Ratio = 7.116



$\alpha = 110^{\circ}$
Rise Ratio = 0.50   Length Ratio = 7.332

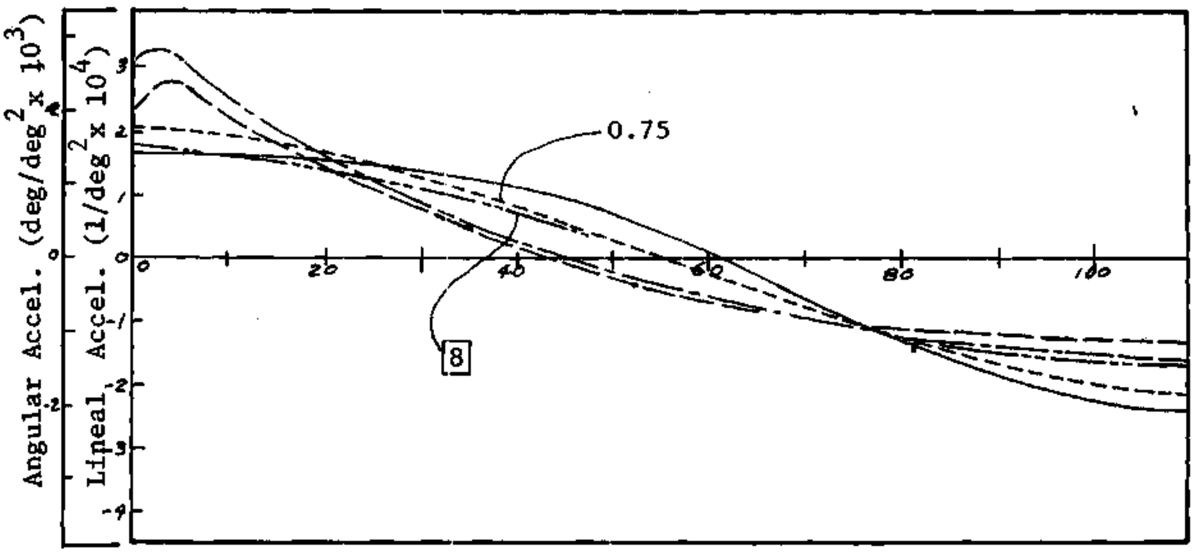
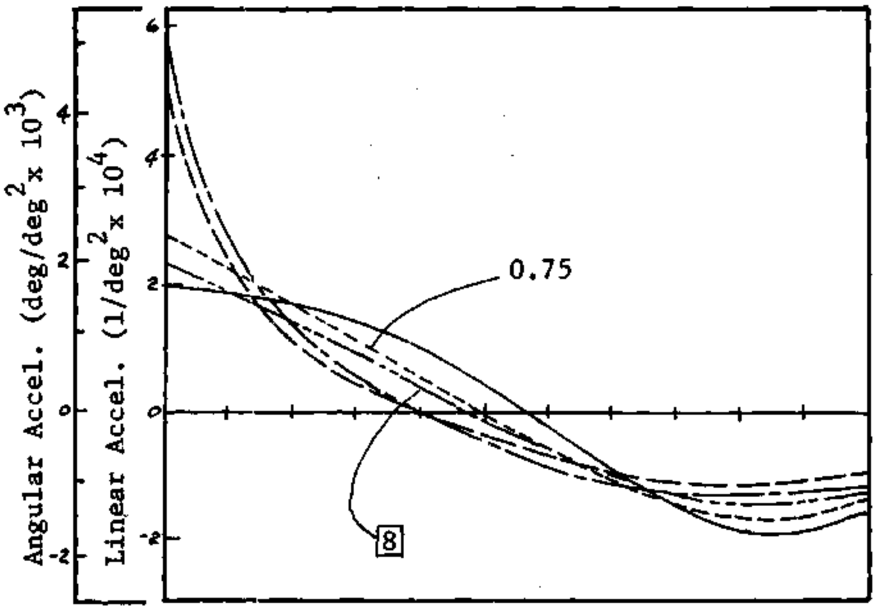


Figure 30. Follower Output Charts

$\alpha = 110^{\circ}$

Rise Ratio = 0.50 Length Ratio = 7.372



$\alpha = 130^{\circ}$

Rise Ratio = 0.50 Length Ratio = 7.226

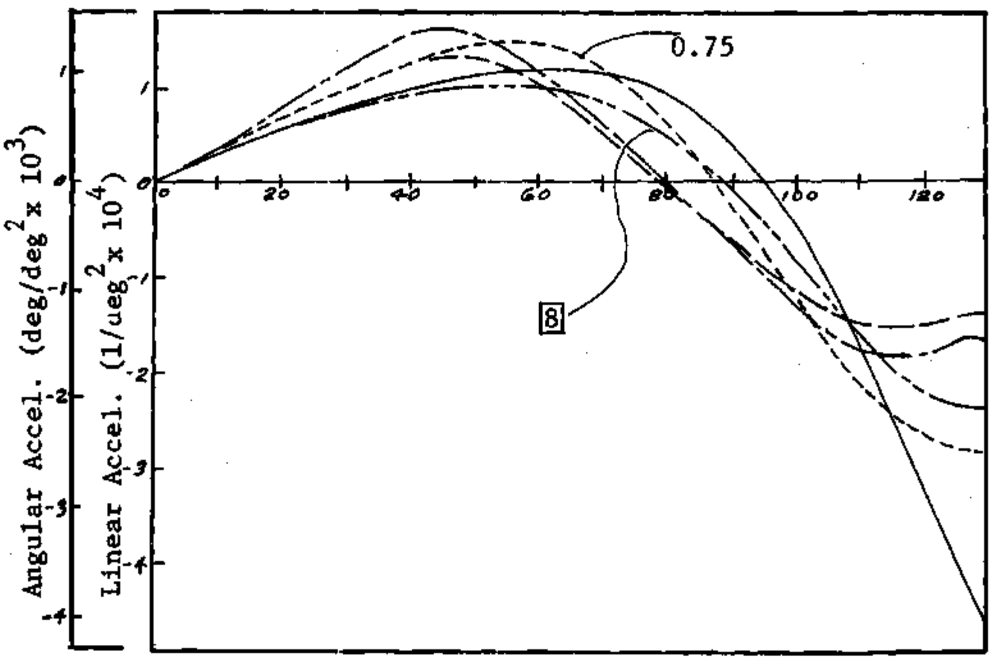


Figure 31. Follower Output Charts

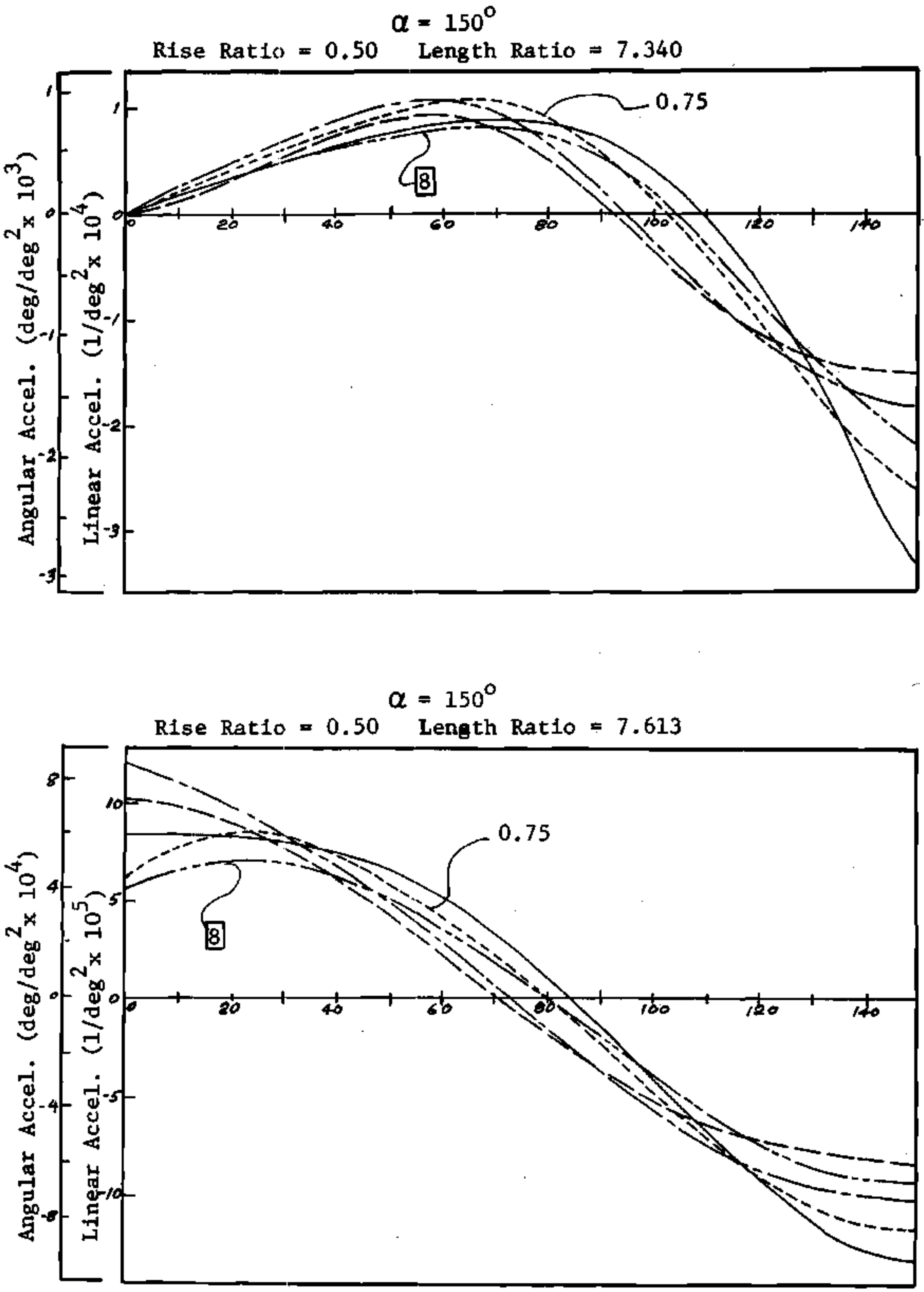


Figure 32. Follower Output Charts

A very noticeable effect of a follower operating on an elastic band cam is that it tends to increase the maximum positive acceleration and decrease the maximum negative. This is important because many of the elastic band cams have a very high value of the maximum negative acceleration compared to the maximum positive acceleration.

Comparison with Conventional Cam Profiles

The elastic band cam can be compared with conventional cams such as the parabolic, simple harmonic, or cycloidal motion by comparing maximum acceleration values and noting the presence or absence of infinite jerk. These types of cam motions are not necessarily the best available but they are well known; comparing the elastic band cam to them gives results which can easily be related to other types of cams. The comparison will be made among dwell-rise-return-dwell cams that have the same rise in a given angle of cam. The shape of the displacement curve of the elastic band cam is dependent upon the size of the base circle whereas the shape of the displacement curve of a conventional cam is not. For the sake of comparison, a base circle radius of 0.500 inch will be used. This will result in a relatively small cam. Larger elastic band cams will have lesser values of acceleration than these cams. Therefore, the comparisons will be on the conservative side. Values for translating roller and translating flat-faced followers are given.

Figure 33 gives a comparison of the displacement and acceleration curves for translating roller and flat-faced followers operating on an elastic band cam with simple harmonic and cycloidal motion cams. The elastic band cam used for the illustration has a continuous accel-

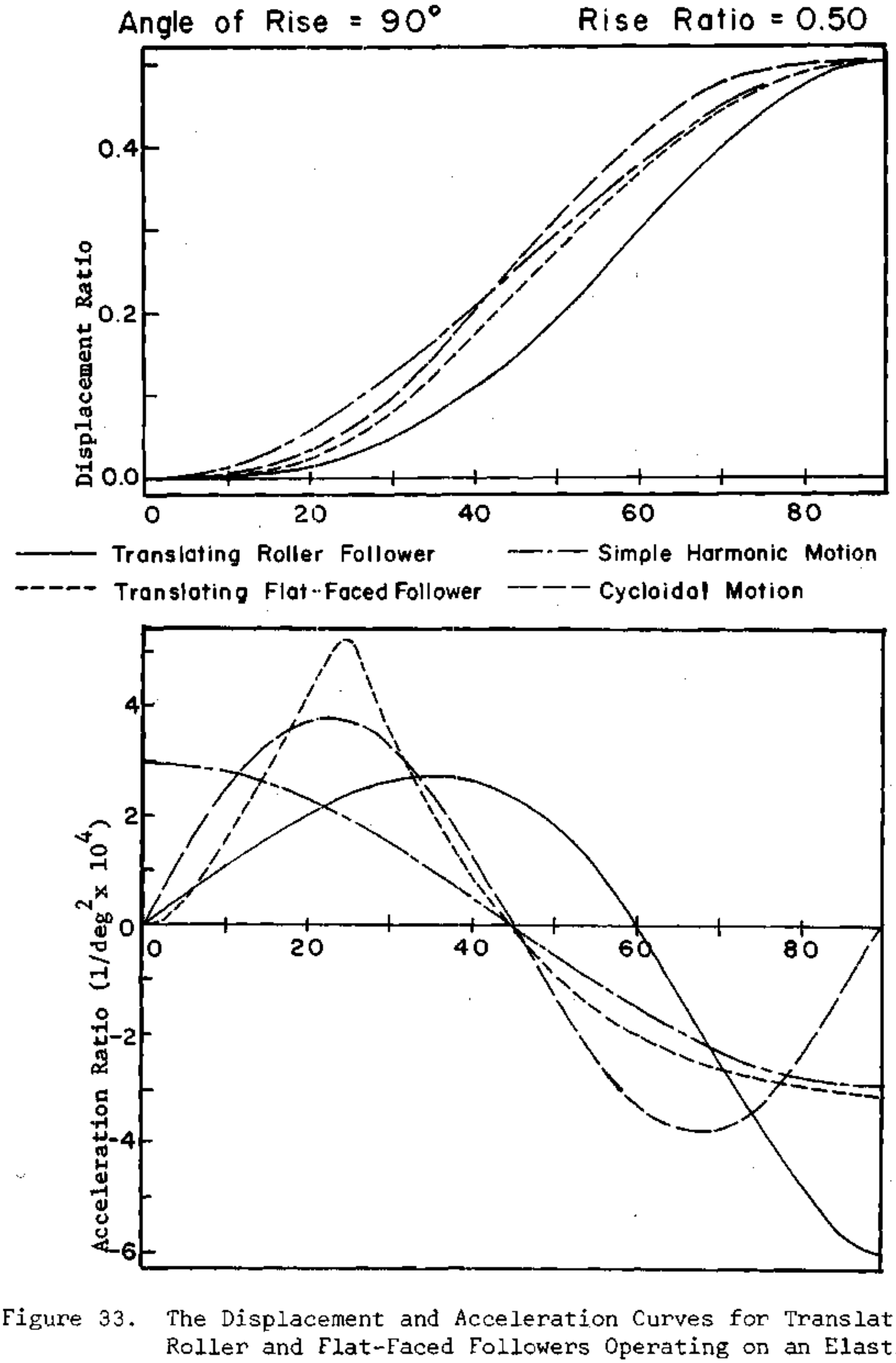


Figure 33. The Displacement and Acceleration Curves for Translating Roller and Flat-Faced Followers Operating on an Elastic Band Cam for Comparison with Simple Harmonic and Cycloidal Motions

eration curve. Further comparisons of the maximum accelerations are made with values taken from the follower output charts and are presented in Table 6.

Table 6. Maximum Acceleration of Parabolic, Simple Harmonic, and Cycloidal Motions and the Maximum Accelerations of Reciprocating Roller and Flat-Faced Followers on Elastic Band Cams

Base Circle Radius = 0.500 in.; Rise = 0.250 in.

Maximum Follower Acceleration Ratio ($1/\text{deg.}^2 \times 10^4$)

				Recip. Roller (.750 Dia.Ratio)			Recip. Flat-Faced		
α	Para.	SHM	Cyc.	(1)	(2)	(3)	(1)	(2)	(3)
30°	22.22	27.41	34.91	+52.2 -12.2	-	-	-	-	-
50°	8.00	9.87	12.57	+27.0 - 7.9			-		
70°	4.08	5.04	6.42	+ 5.2 - 9.0	+9.2 -3.8	+15.0 - 3.2	+19.0 - 3.5	- -	- -
90°	2.46	3.04	3.88	+ 2.6 - 6.0			+ 5.1 - 3.1		
110°	1.68	2.04	2.58	+ 1.8 - 4.3	+2.1 -2.3	+ 2.7 - 1.7	+ 2.2 - 2.7	+3.4 -1.6	+4.8 -1.5
130°	1.18	1.46	1.86	+ 1.4 - 2.8			+ 1.5 - 1.8		
150°	0.90	1.10	1.38	+ 1.1 - 2.6	+0.8 -1.1	+ 1.2 - 0.8	+ 1.1 - 1.8	+1.2 -1.0	+1.7 -0.7

NOTE: When a dash appears it means that this cam cannot be used with this type follower because of the negative curvature of the profile.

(1) Elastic band cam with zero initial acceleration or when this is not possible, the minimum length band.

(2) Elastic band cam with free elastic band.

(3) Elastic band cam with band length longer than for the free case.

In this table only the elastic band type denoted (1) has a continuous acceleration. The other two have a discontinuity at the point of initial rise. For parabolic and simple harmonic motions the jerk becomes infinite, but it remains finite for cycloidal motion. The positive and negative values of maximum acceleration for the conventional cams are equal, but these values differ for elastic band cams. There is no constant relation between maximum acceleration values of different types of cams with the elastic band cams. The comparison depends on the particular cam chosen.

Harmonic Analysis of Acceleration Curves

Additional information to be used in the evaluation of the elastic band cams is obtained from a harmonic analysis of the cam and follower acceleration curves. The method of calculation of the constituent harmonics is presented in terms of the cam acceleration r" versus Θ, but the procedures are the same for the follower acceleration. Each curve is represented by a set of precision points (Θ_i, r''_i) and can be defined by a finite trigonometric sum. Let the set of precision points:

$$(\Theta_o, r''_o),\ (\Theta_1, r''_1),\ (\Theta_2, r''_2),\ \cdots\ (\Theta_{2n-1}, r''_{2n-1}),\ (\Theta_{2n}, r''_{2n})$$

be such that the values of r''_i, the acceleration, start repeating with r''_{2n}, that is, $r''_{2n+1} = r''_i$. The Θ_i must be equally spaced over a period equal to the period of the cam. The trigonometric polynomial:

$$r'' = A_o + \sum_{k=1}^{n} A_k \cos\left(\frac{k\pi\Theta_i}{\alpha}\right) + \sum_{k=1}^{n} B_k \sin\left(\frac{k\pi\Theta_i}{\alpha}\right) \qquad (3\text{-}29)$$

represents r'' as a function of Θ when the constants A_i and B_i are determined. These are found from the following equations:

$$A_o = \frac{1}{2n} \sum_{i=o}^{2n-1} r''_i$$

$$A_j = \frac{1}{n} \sum_{i=o}^{2n-1} r''_i \cos\left(\frac{j\pi\Theta_i}{\alpha}\right) \quad \text{for } j = 1,2,\cdots,(n-1)$$

$$= \frac{1}{2n} \sum_{i=o}^{2n-1} r''_i \cos\left(\frac{n\pi\Theta_i}{\alpha}\right)$$

$$B_j = \frac{1}{n} \sum_{i=0}^{2n-1} r''_i \sin\left(\frac{j\pi\Theta_i}{\alpha}\right) \quad \text{for } j = 1,2,\cdots,(n-1)$$

The constituent harmonics of some of the cam and follower acceleration curves were calculated so that they are available for use in vibration analyses of the cams. The acceleration curves are even functions, and therefore the coefficients of the sine terms of the Fourier expansion representing the curve are zero. The expansion is of the form:

$$r'' = A_o + A_1 \cos(\Theta) + A_2 \cos(2\Theta) + \cdots + A_n \cos(n\Theta) + \cdots$$

The term A_o is the average value of the acceleration. This is zero because the net area under the acceleration curve equals zero for a cam. For the cam accelerations only the first four harmonics, Fourier coefficients, A_n, were generally found to be of significant magnitude to be considered and plotted (Figures 34 and 35). The higher harmonics are greatest in amplitude for cams which have a zero initial acceleration. This is because these cams have the greatest differences between the maximum positive and maximum negative accelerations. With the exception of A_1 which is almost a constant, the amplitudes of the harmonics were a minimum for cams having almost no external loading on the band, that is approximately the case of the free elastic band cam. In Figure 35, graphs of the constituent harmonics of the acceleration curves of cams with free elastic bands are plotted.

The constituent harmonics were plotted for two types of follower systems: A translating roller follower with a roller radius ratio equal 0.75 and zero offset (Figure 36) and a translating flat-faced follower (Figure 37). These charts are very similar to the charts for the constituent harmonics of the cam acceleration. The amplitudes of the first harmonic of the follower accelerations are almost a constant and approximately equal to the first harmonic of the cam profile on which the follower operates. For the roller follower the other harmonics have their maximum amplitudes for the case of the cam with a continuous acceleration. The minimum amplitudes of the harmonics occur for a length of the band somewhere between the lengths for the continuous acceleration and the free elastic band cams. For the flat-faced follower, the maximum amplitudes of the harmonics occur at the minimum length of band and at the maximum band length.

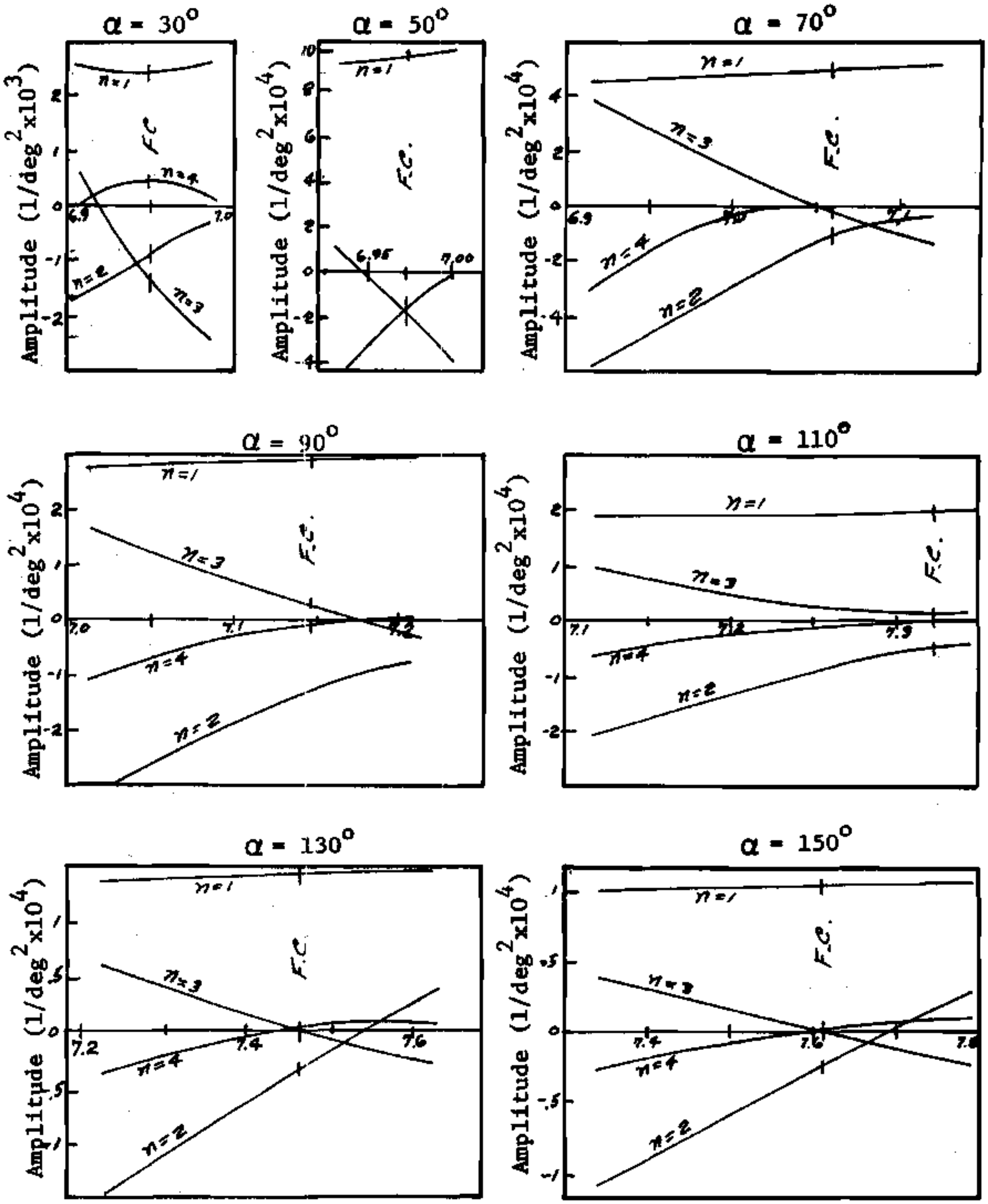


Figure 34. Constituent Harmonics of Cam Accelerations
Plotted versus Length Ratio

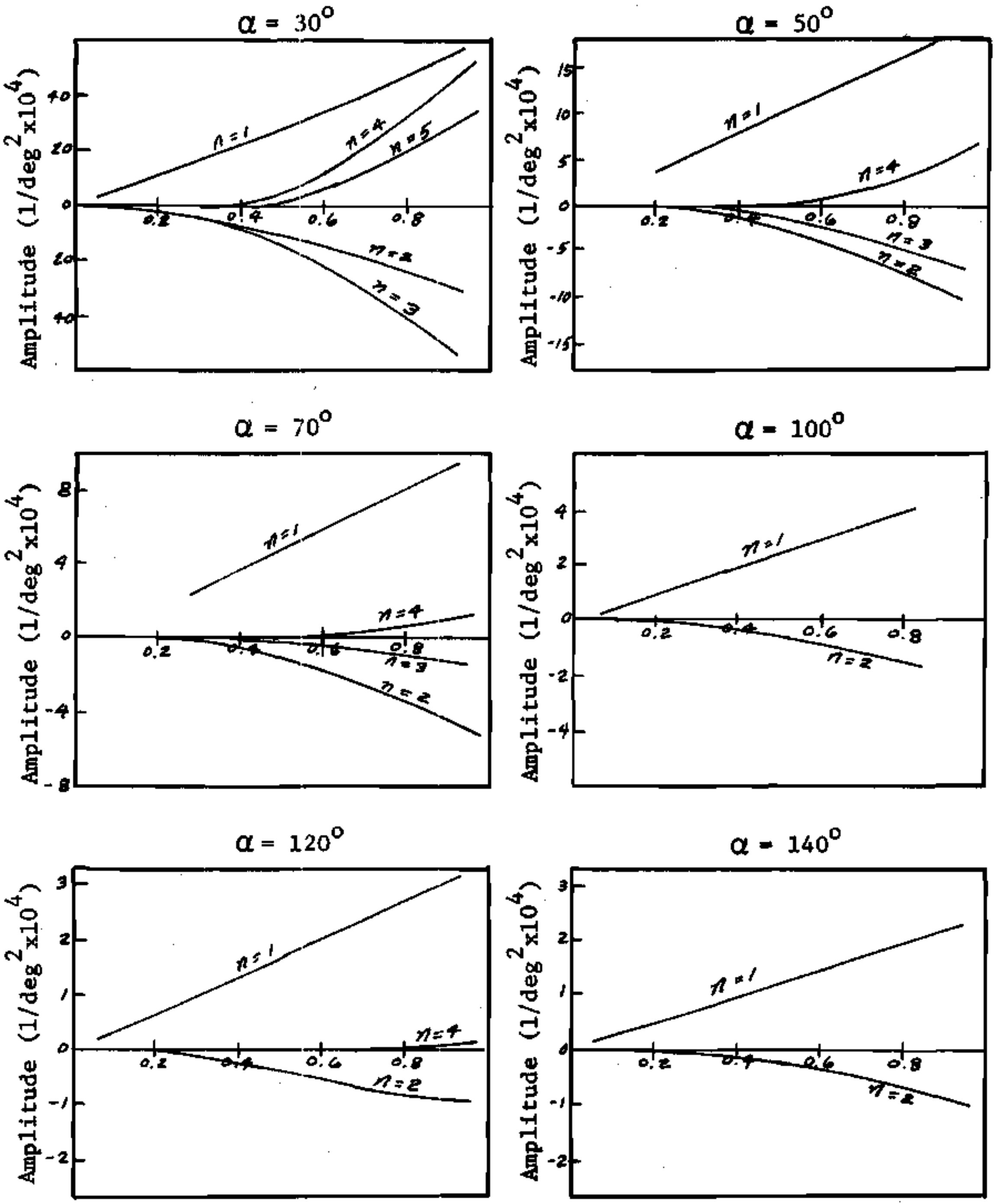


Figure 35. Constituent Harmonics of the Accelerations of Cams with a Free Elastic Band Plotted versus Rise Ratio

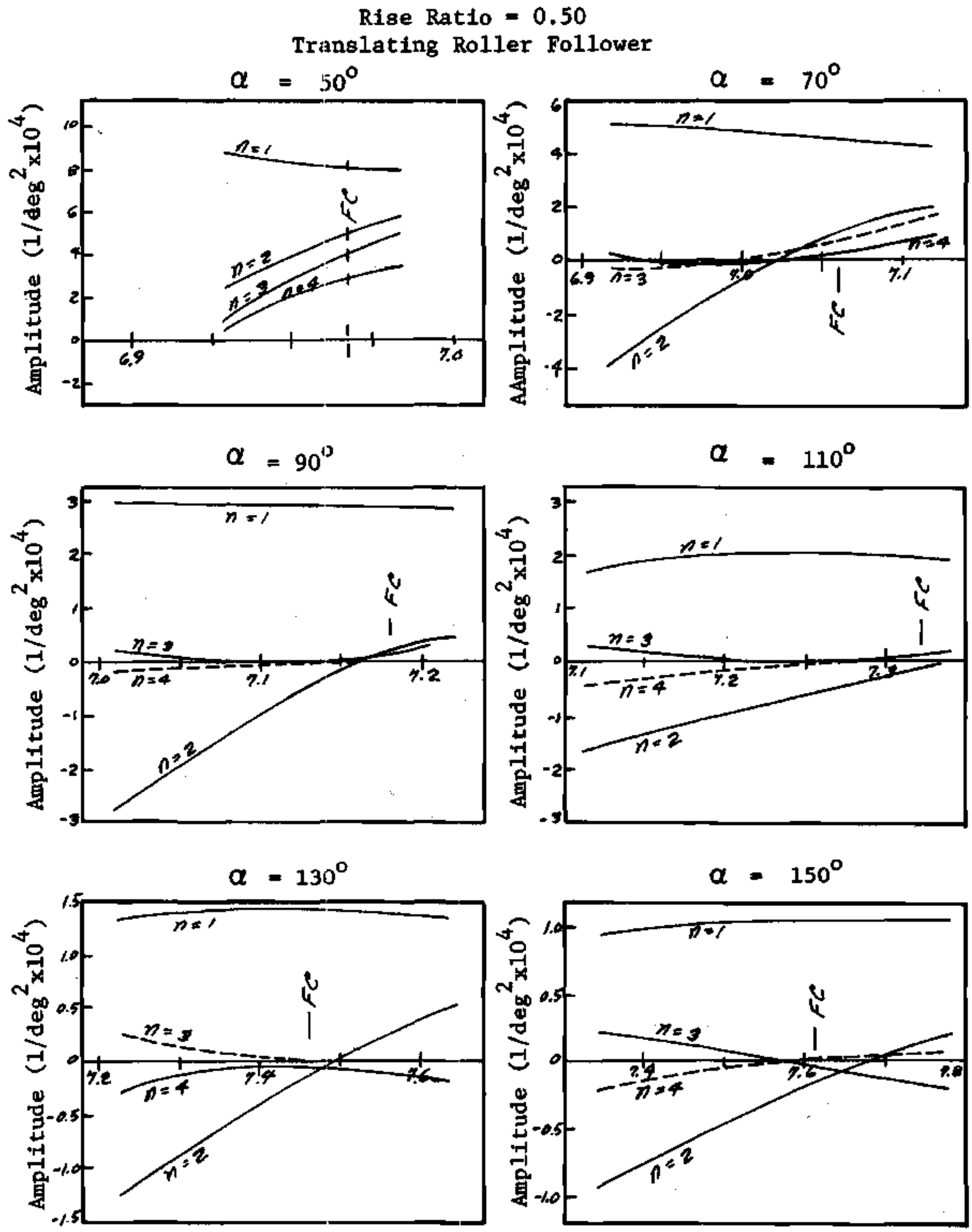


Figure 36. Constituent Harmonics of Acceleration Curve for a Translating Roller Follower Plotted versus Length Ratio

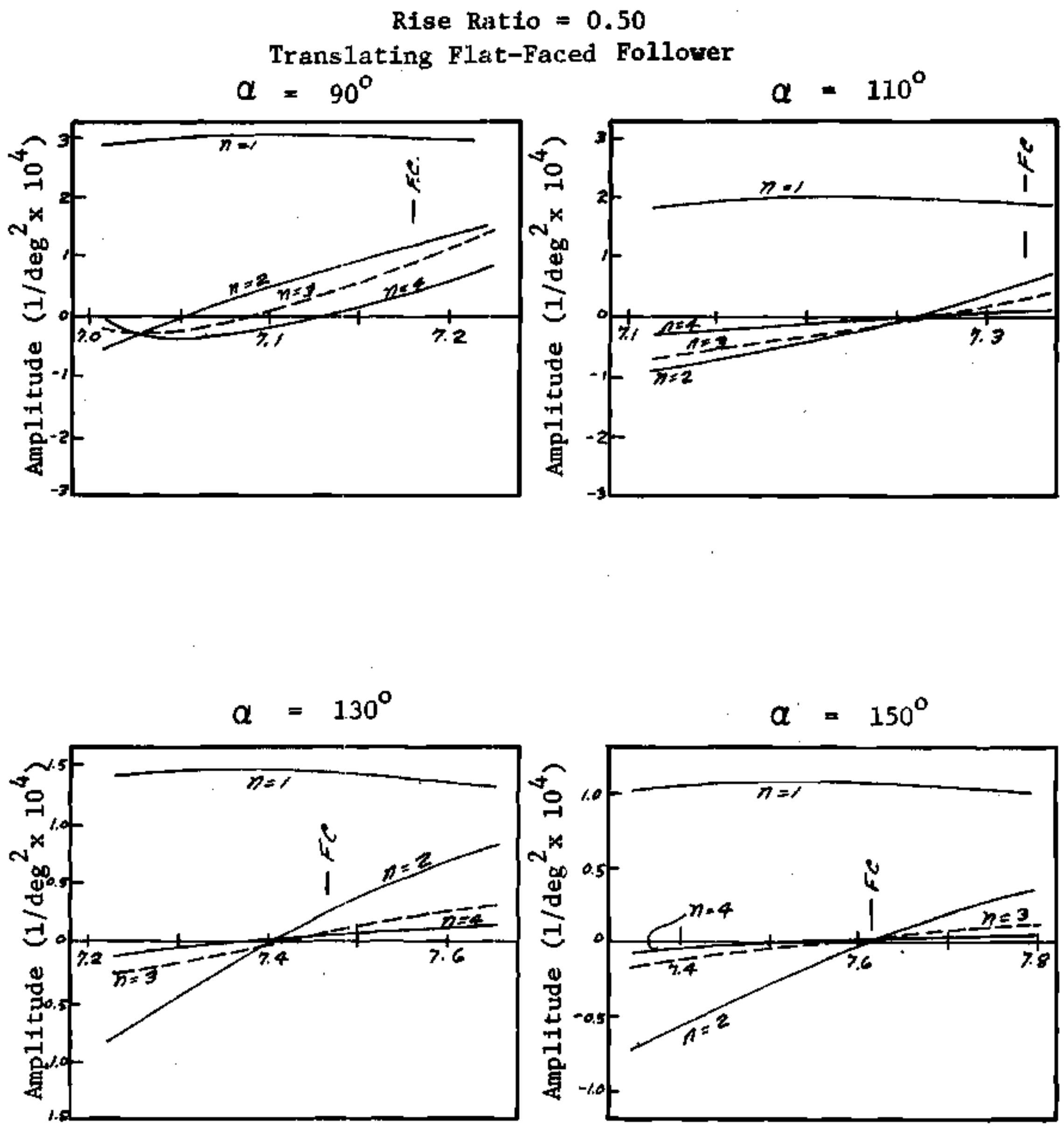


Figure 37. Constituent Harmonics of Acceleration Curves for a Translating Flat-Faced Follower Plotted Versus Length Ratio

The Approximate Dwell-Rise-Dwell Configuration

Another configuration of the elastic band cam which seemed to have possible practical uses is the case where the band passes around a circular pin at the point of maximum rise (Figure 38). By making the radius of the pin large, an approximate dwell-rise-dwell cam can be formed. The solution and therefore any tabulation of results for this cam involves five parameters:

(1) the angle from point of contact with hub to cam center line (α),

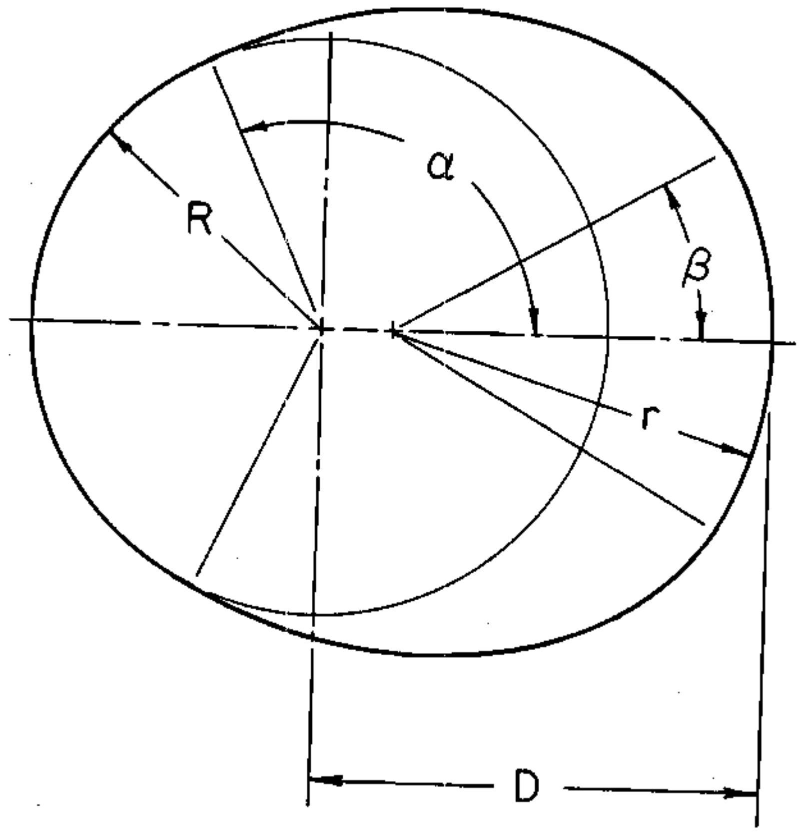
(2) the angle of wrap around the pin (β),

(3) the ratio of pin radius to hub radius (r/R),

(4) the rise ratio (D-R/R), and

(5) the length ratio (L/R).

A cam which is designed by this method will always have finite discontinuities in the acceleration. If the length ratio is adjusted so that the discontinuity is eliminated at the point of contact with the hub, the discontinuity at the point of contact with the pin increases (Figure 39). If the length ratio is adjusted to eliminate the discontinuity at the pin, the discontinuity at the hub increases. Therefore this configuration of elastic band cam does not have good dynamic characteristics and was not investigated further.



α = Angle from Point of Contact to Line of Symmetry

β = Angle of Wrap of Band Around Circular Pin

R = Radius of Cam Base Circle

r = Radius of Circular Pin

D = Distance from Cam Center to Toe

Figure 38. Cam Configuration with Circular Pin at the Point of Maximum Rise

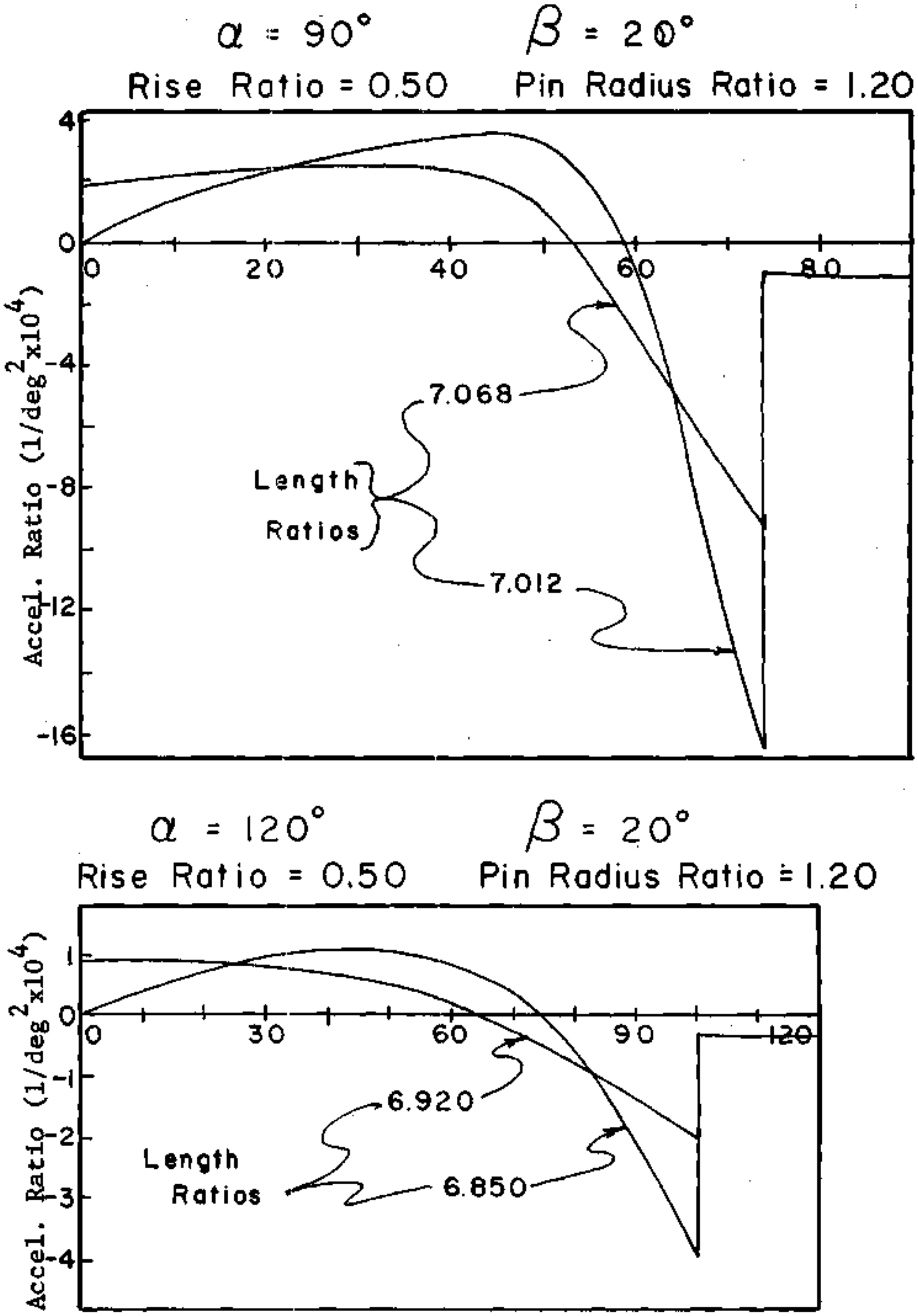


Figure 39. Example Data of Dwell-Rise--
Dwell Elastic Band Cams

CHAPTER IV

EXPERIMENTAL INVESTIGATION

Although the analytical investigation of the elastic band cam comprises the greater part of this study, a few sample cams are made and tested. The purpose of this part of the work is to verify experimentally the mathematical representations of the cams and the feasibility of fabricating cams of this type.

Fabrication of Cams

The fabrication of elastic band cams is developed only to the point that it is adequate for the verification tests made. First, a solid hub of steel is bored to fit snugly on the camshaft of the test jig (Figure 40). The outside diameter of the hub is turned to a dimension equal to the nominal size of the base circle of the cam minus twice the thickness of the band. The completed cam then has the correct base circle diameter. Next the band is put in place and held by the clamps. The void between the band and the hub is filled with a matrix material. This holds the band in the correct position when the restraining forces are removed and also gives the band enough rigidity to support the follower loads.

A holding fixture is built to hold the cam during fabrication (Figure 41). The center peg is used to locate the hub and is the same size as the shaft on which the cam will be mounted. Six screws are used

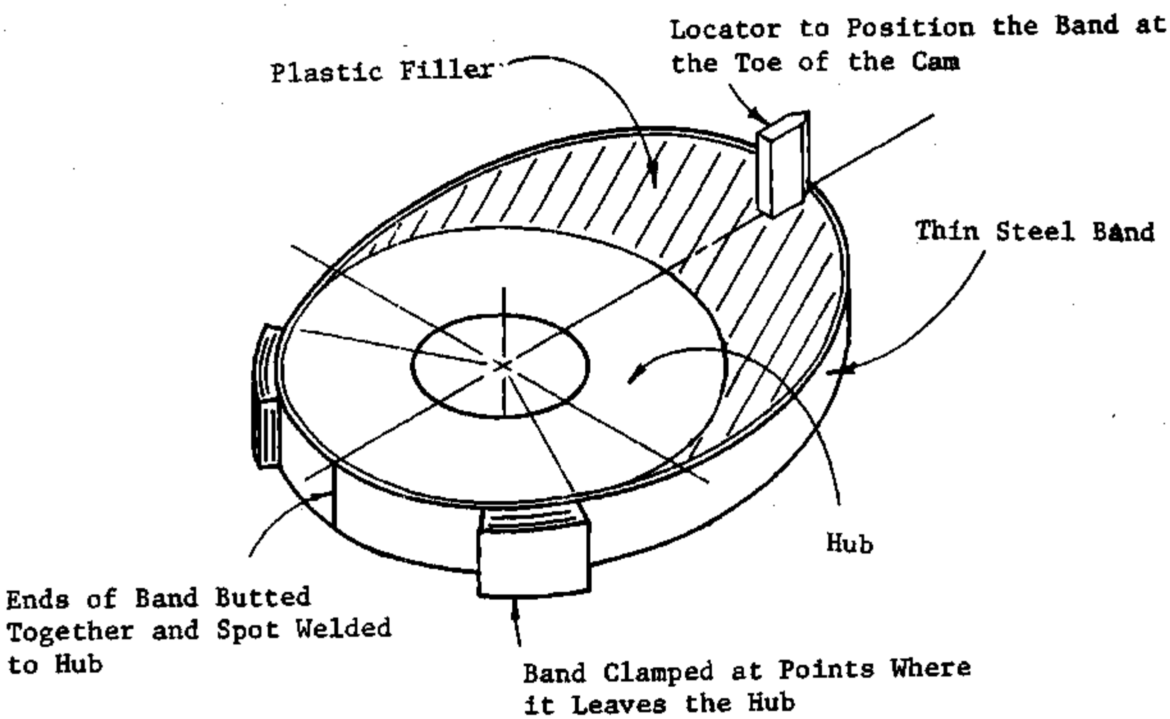


Figure 40. An Elastic Band Cam

to apply pressure to the clamp blocks so that the blocks hold the elastic band to the hub around the heel of the cam. The clamp blocks have an inside radius that matches the base circle radius of the cam. As is shown in the insert in Figure 41, one block on each side of the cam has a square corner which is aligned with the location of the point of contact between the band and hub giving the required fixed end effect at those points. The band locator is used to position the band at the point of maximum rise when this is required by the design. The locator pin can be turned so that its knife edge can be used to apply a force either toward or away from the cam center. Both the clamp blocks and the band locator are positioned by aligning one of their edges with a scribed line on the base of the fixture.

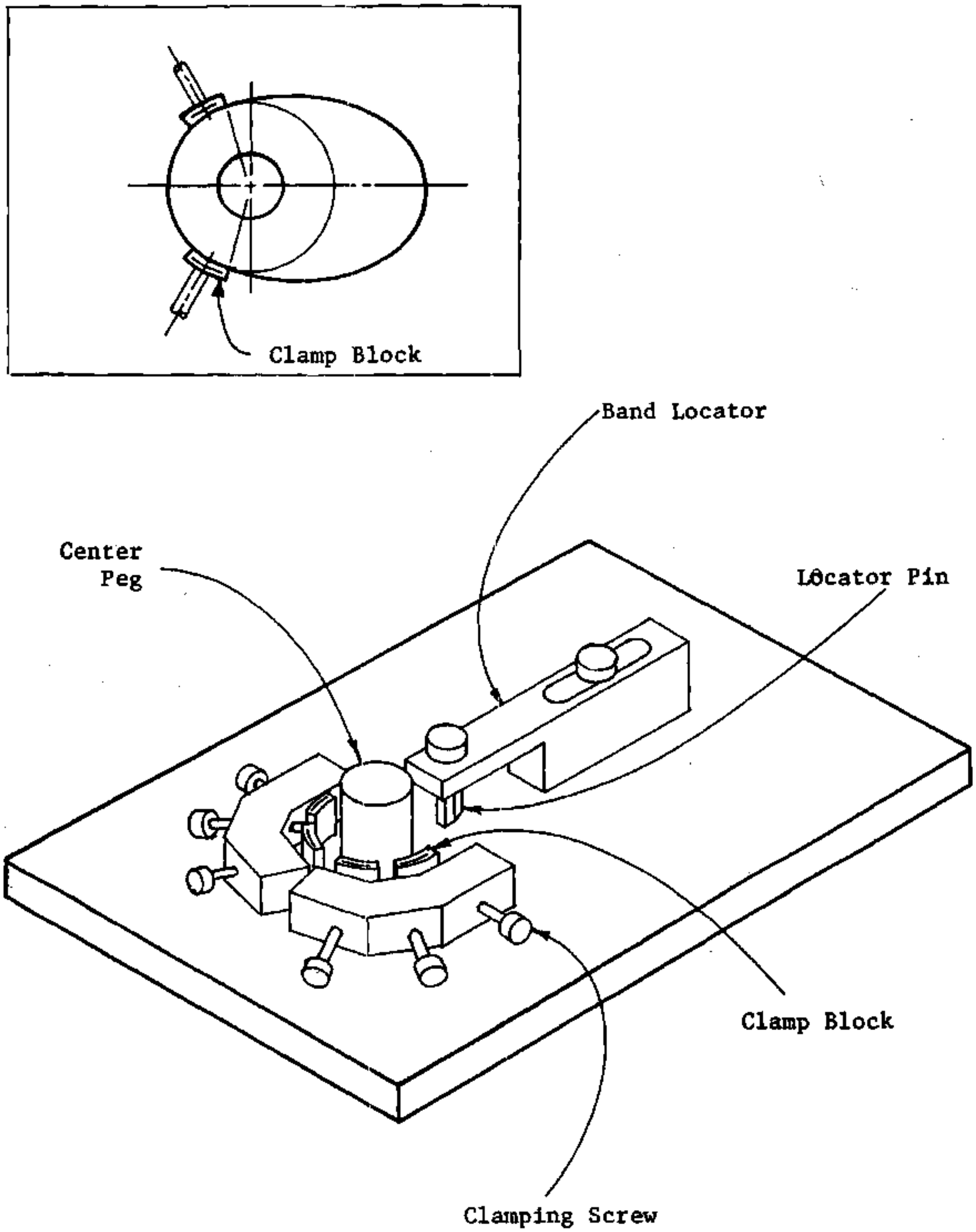


Figure 41. Cam Holding Fixture

The void between the hub and band is filled with the matrix material while the cam is still in the holding fixture. After the matrix sets and the cam are removed from the holding fixture, the band is spot welded to the hub at points along the heel with a strain gauge spot welder. The ends of the band are welded along the joint and more of the filler material added to fill any irregularities in the joint. The surface of the joint is then polished smooth.

For the band a strip of steel shim stock 1/4 to 3/8 inch wide is used. The best thickness of this band depends on the size of the cam. The band should be as thick as possible without being so thick that stresses in the band exceed the elastic limit and cause plastic deformation. A band 0.003 inch thick is used to make cams with base circles radii from 0.500 to 0.750 inch.

Any gap between the abutting ends of the band, or error in cutting the length of the band must of course affect the accuracy of the cam. A gap where the ends are joined has the same effect as lengthening the band. Since the rise and angle of rise are set independently of the band length, these are not affected. An example will be used to illustrate the effects of an error in the band length. For a cam, $R = 0.50$, $D = 0.75$, $\alpha = 110°$, and $L = 3.600$, reference to Figure 22 for $\alpha = 110°$, and length ratio 7.200, shows that an error increasing the band length decreases the peak acceleration and vice versa. The change in peak acceleration per change in length is proportional to the slope of the line for maximum negative acceleration. This is $20 \frac{\text{in/deg}^2}{\text{in}}$. The cam displacement as measured from the cam center is also affected by the error in length. Using the computer, the displacement curve was calculated for the cam R = 0.500, D = 0.750, α = 110°, and L = 3.638. Then the displacement curve when the length of the band was 0.005 longer or shorter than initially specified has been calculated for comparison. From these curves the change in cam displacement per change in band lengths at several points was obtained.

Cam Angle	Cam Displacement L = 3.638	Cam Displacement L = 3.643	Change in Displacement
30°	0.5312	0.5324	0.00024 in/0.001 in
60°	0.6183	0.6209	0.00051 in/0.001 in
90°	0.7209	0.7222	0.00025 in/0.001 in

The 0.51 in/in. change is approximately the maximum in this case. It should be noted, however, that a band length error does not disturb the continuity of the profile as would a similar error in conventional manufacturing methods, and therefore it is not as serious.

For the matrix an epoxy plastic marketed under the trade name Epoxybond by Atlas Minerals and Chemicals Division of the Electric Storage Battery Co. was chosen. The modulus of elasticity is approximately 0.7 x 10^6 psi. It has a compressive strength of 9,000 psi. The advantages of this epoxy resin include its ability to form very good bonds with the steel band and hub, and that it is very easily handled and cured without any special equipment. Measurement showed that the midpoint of the side of a cam, R = 0.500, D = 827, α = 86, and L = 3.615 shrinks 0.61 of 1 per cent during curing. This was measured at the point of maximum shrinkage; at all other points it is less. The resulting change in the displacement of this cam at this point is 0.0013 in. This will slightly increase the peak positive and negative accelerations.

The epoxy plastic used is a good, but not necessarily the optimal,

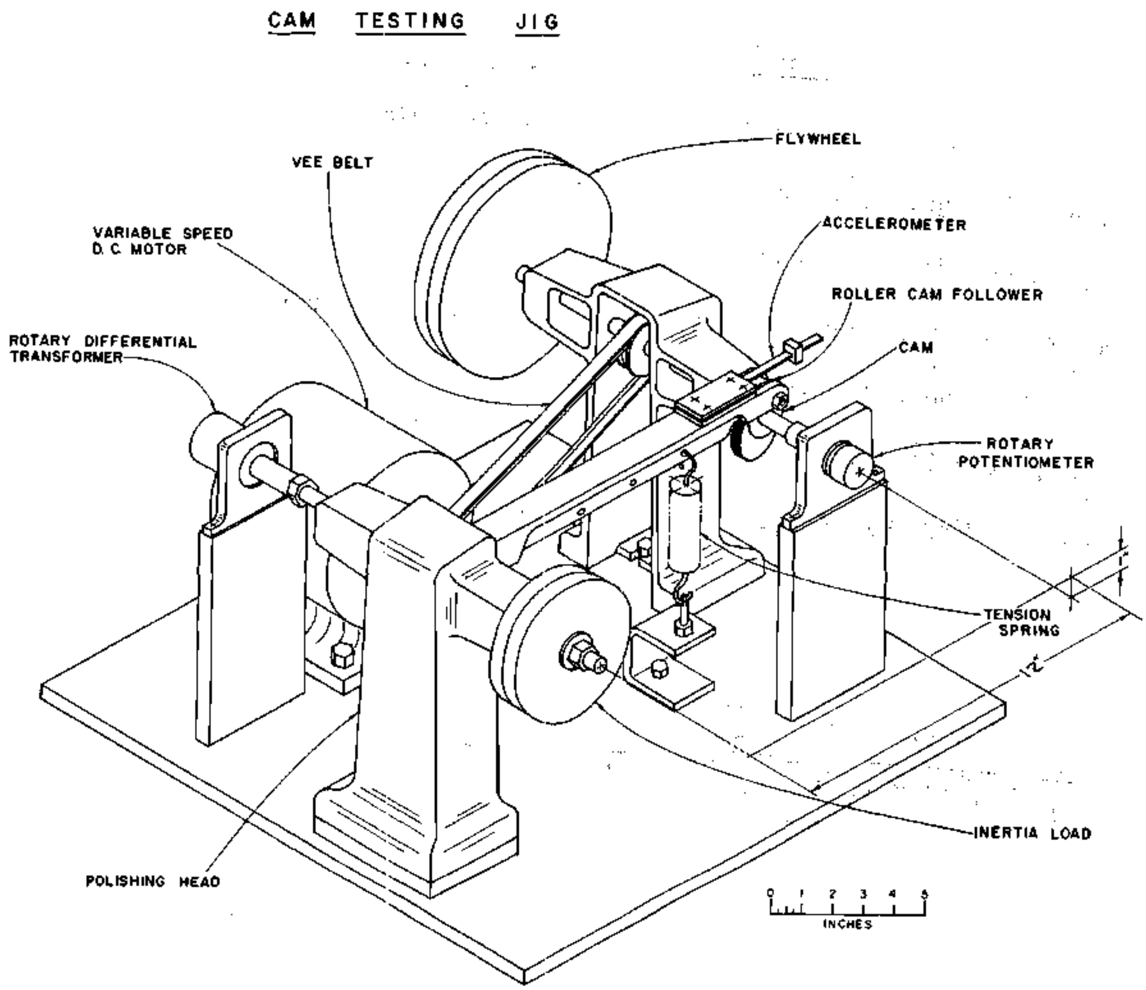


Figure 42. Cam Testing Jig

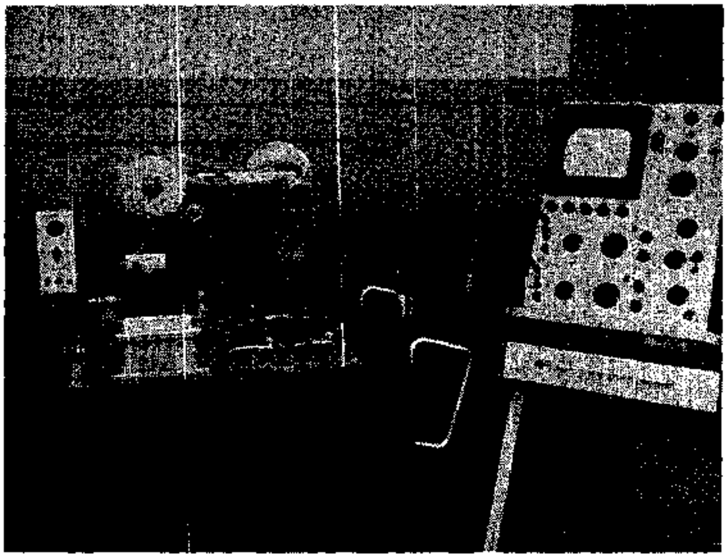


Figure 43. Cam Test Jig and Supporting Equipment

choice for a matrix material. With other epoxy resins there would not be a great variation in the physical properties, but the casting characteristics might be better. A metal matrix which might be used is an eutectic alloy of Lead and Bismuth (44.5 per cent Pb, 55.5 per cent Bi) which has the trade name Cerrobase. It has a melting point of 255°F, is very fluid when molten, and, most important, has zero shrinkage during freezing. The modulus of elasticity of this metal is 2.5×10^6 psi and its compressive strength is 13,000 psi.

Another possible inaccuracy of cam manufacture results from the spring-back of the elastic band when the force on the locator at the toe is removed. Experimentally, the change in the toe displacement is measured to be 0.002 inch on a cam R = 0.500, D = 0.875, α = 86°, and L = 3.615. This will cause a decrease in the rise equal to this amount.

However, allowance can easily be made in the initial setting of the locator for the spring-back, so that the final rise is the correct amount. A secondary effect, which cannot so readily be compensated, is that as the toe springs back, so also the radius of its curvature decreases. Since it is at the point that the curvature is the greatest, the spring-back has the desirable effect of reducing the acceleration at that point.

The cams actually fabricated were not intended to be of high precision but accurate enough to be used for the verification work. The band length and the position toe locator was held within ±0.005 inch. The clamps setting the angles of contact with the base circle were accurate within ±1/4 degree. The spring-back at the toe of the cam is less than the tolerances to which the toe locator can be set, but some allowance was made for it by setting the toe locator on the high side of the tolerance range. The cams made proved satisfactory for the test made.

Experimental Work

Equipment

A cam test jig was designed and built to test the sample cams. As shown in Figure 42, the jig was designed with two horizontal shafts, one supporting the cam and the other supporting a radial-type follower arm. The commercially available items are as follows:

1 Cam Yoke Roller, Smith Bearing Co., CYT-21, 3/4 in. Dia.

2 Polishing Heads, Sears, Roebuck, and Company

1 1/8 Hp. D.C. Motor, Bodine, NSH-54

1 D.C. Motor Controller, Minarik No. SH-53

1 3/8 in. Vee Belt

2 Resistance Strain Gauges, BLH Corp. SR-4, Type A-7

1 Precision Potentiometer, Helipot, Model 5433, 1000 ohm

1 Rotary Variable Differential Transformer, Schaevitz Engineering, Type R3B1S

1 Two-Channel Recorder, Sanborn, Model 296

The test jig was constructed in the Mechanical Engineering Department shop. The cam shaft speed is regulated with the variable speed control. An oscillating follower is used rather than a linear translating type because of the bearing problems associated with a linear type. An arm of sufficient length was used so that the motion of the oscillating follower system approximates a linear follower system. The arm is designed so that either a commercial roller follower or a flat-faced follower can be used for the sake of flexibility. The design of the jig also allows the follower spring constant to be varied by changing the position of the spring along the follower arm. The initial spring load at each location is also adjustable.

The rotary potentiometer is used to measure the camshaft position. It is a type which permits 360°, continuous turning. The rotary differential transformer is mounted on the follower pivot shaft and is used to measure the angular displacement of the follower. The range of operation, the range of follower displacements, is only of the order of ±2.5 degrees when the cam rise is 0.50 inch. Since the rotary differential transformer output is linear within ±0.1 per cent for ±10 degrees of rotation, nonlinearity is no problem.

A strain gauge accelerometer was designed and built. A commercial accelerometer was not used because the frequency of the accelerations, the camshaft speeds, are too low to excite accelerometers of normal commercial designs. The accelerometer which was designed had a much lower natural frequency than commercial types. Therefore, the exciting forces will produce much higher amplitude responses. The basic design consists of a cantilever beam of ground steel stock with a small moveable weight attached near the free end. The acceleration of such a system is proportional to the deflection of the mass of the system. The steep strip was chosen to be of 0.0325 inch stock, 0.500 inch wide, and the weight to be 0.0055 lb., because then the frequency range would be within the range 70 and 140 cps. It was arranged that the length of the beam, and the location and magnitude of the weight could be adjusted to obtain the best possible results from the accelerometer. The dimensions of the accelerometer were set as follows: The beam is 2.25 inches long and the weight is attached 1/2 inch from the free end. The output from this system proved adequate for the verification tests undertaken.

An equivalent system for the accelerometer is formed (Figure 44). Since the actual spring, the beam, is not weightless, half of its weight is included with the attached mass and the other half with the fixed support. The mass of the equivalent system equals one half the mass of the beam plus the mass of the moveable weight. The length of the equivalent spring is calculated so that the moments on the spring equal the moments on the actual beam. The moment arm of the equivalent system is:

$$Z = \frac{1\ P + \frac{3}{8}\,wL^2}{P + w\,\frac{L}{2}} \qquad (4\text{-}1)$$

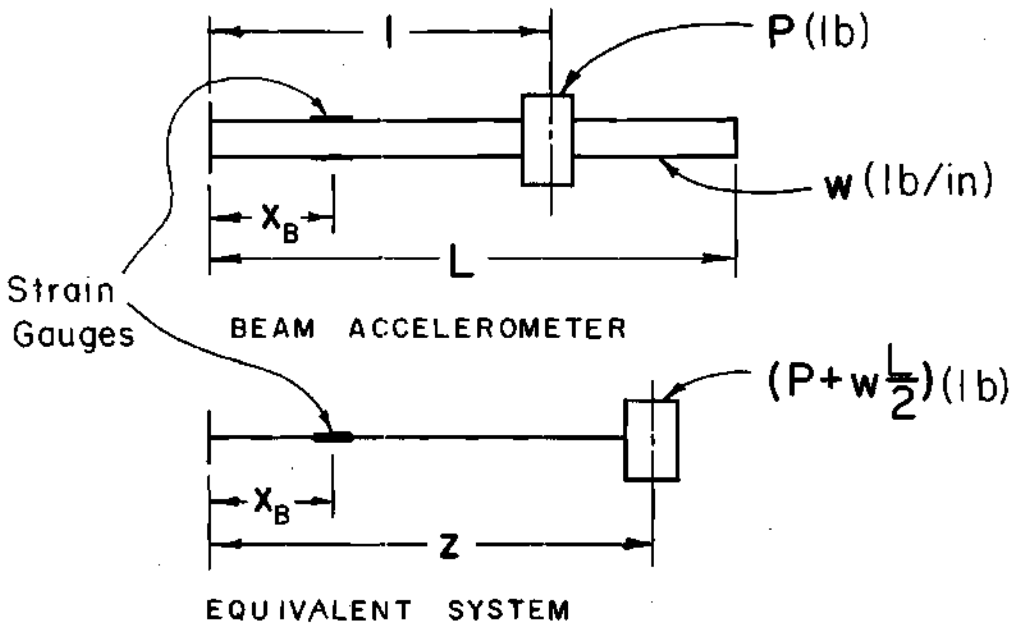


Figure 44. Diagram of Strain Gauge Accelerometer, and the Equivalent System of the Accelerometer

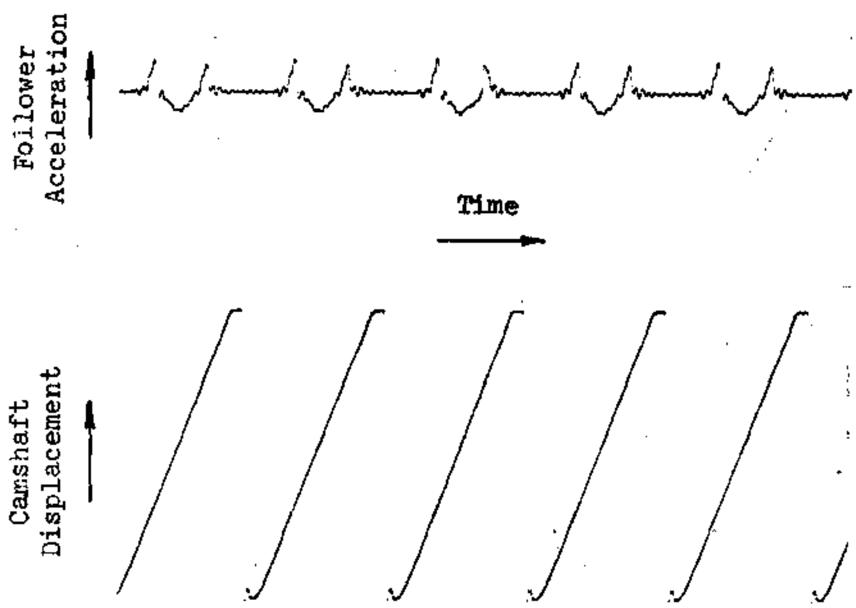


Figure 45. Example Accelerometer Output and Input Angular Displacement of Cam versus Time

Since the strain gauges on the accelerometer measure the bending moment; the deflection is calculated from it for the equivalent system from the equation:

$$M_B = -(P + w \frac{L^2}{2})(Z - x_B) \tag{4-2}$$

The deflection of the equivalent system at the free end is:

$$y = - \frac{(P + w \frac{L}{2})\ Z^3}{3\ EI} \tag{4-3}$$

The section modulus of the equivalent system is the same as for the actual beam. Combining Equations (4-2) and (4-3), we obtain the deflection of the mass as a function of the bending moment:

$$y = \left[\frac{Z^3}{3(Z - x_B)\ EI}\right] M_B \tag{4-4}$$

The expression as given by Macduff and Curreri (60) relating the acceleration and the deflection is:

$$a = \frac{y}{\omega_n^2 \sqrt{[1 - (\omega/\omega_n)^2]^2 + [2r\ \omega/\omega_n]^2}} \tag{4-5}$$

ω = frequency of exciting force

ω_n = natural frequency of accelerometer

r = damping factor (c/c_c)

According to Macduff an accelerometer should be designed so that the ratio of the exciting frequency to the natural frequency of the accelerometer (ω/ω_n) is less than 0.10. The cams are tested at speeds

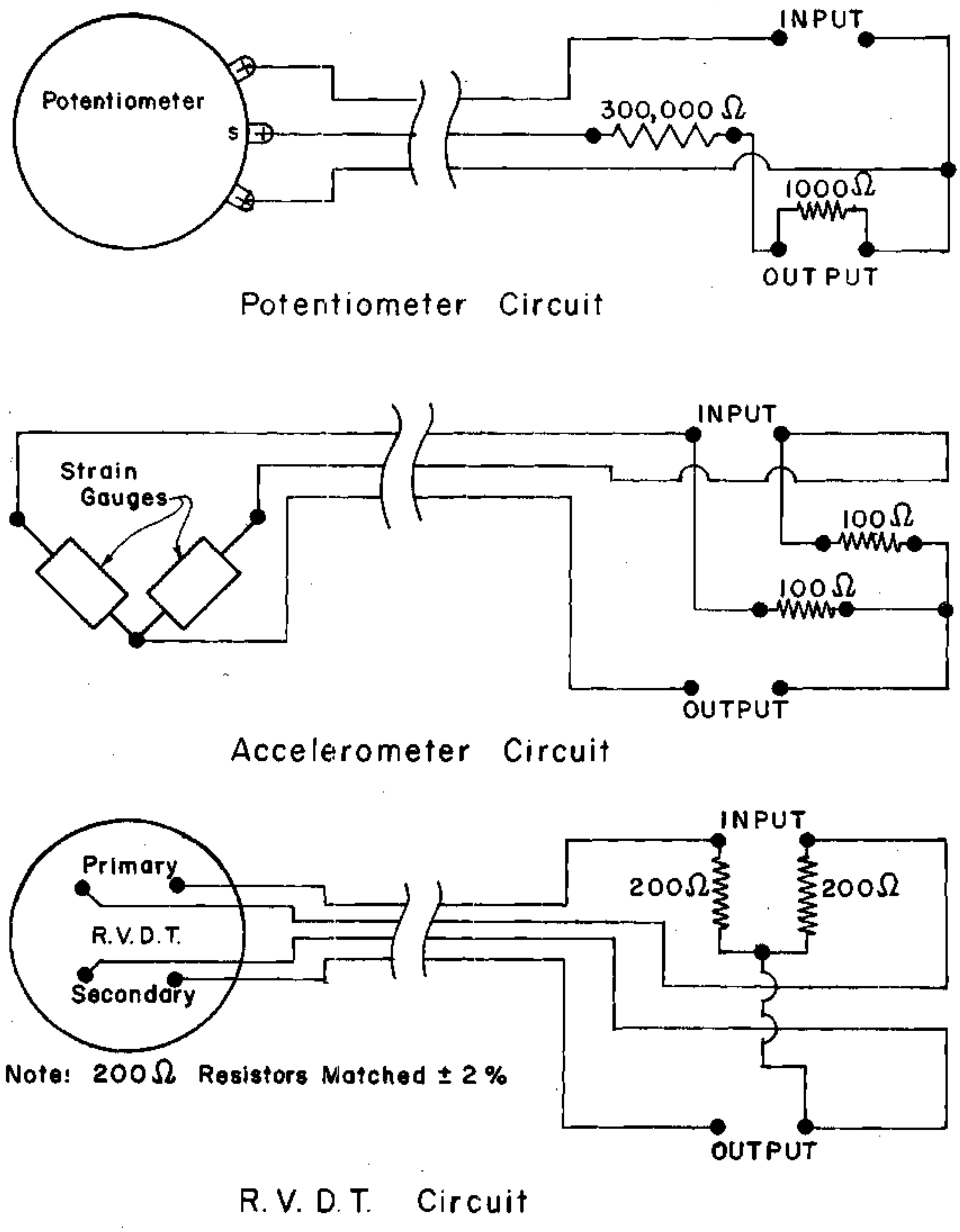


Figure 46. Circuits for Connecting Transducers to the Sanborn Recorder

between four and six revolutions per second; the natural frequency of the accelerometer was experimentally measured to be 111 cps.; the ratio of frequencies is thus under 0.06. The damping factor was measured by checking the logarithmic decrement and found to be 0.084. This is not near the optimal value of 0.7, but since the ratio of ω/ω_n for all test runs is less than 0.1, the low damping factor does not adversely affect the results. The plot of the bending moment in the beam of the accelerometer allows the acceleration of the its point of attachment to the follower arm to be calculated. The output from the accelerometer (Figure 45) is a curve in which the desired result, the acceleration, is mixed with the natural frequency of the accelerometer.

In order to obtain readings from the transducers, they are connected to a Sanborn recorder through simple circuits (Figure 46). The input voltage which is required for the operation of the measuring devices is supplied from the preamplifier of the Sanborn recorder. The output from the instruments is fed back into the Sanborn and recorded on moving chart paper.

Test Procedures

The procedure for testing a cam involves two basic steps:

(1) measuring the cam displacement and producing a plot of the displacement of the follower versus the angular displacement of the camshaft and

(2) measuring the acceleration of the follower and producing a plot of the follower acceleration versus the angular displacement of the camshaft.

To measure the displacement, the cam is run at a low speed.

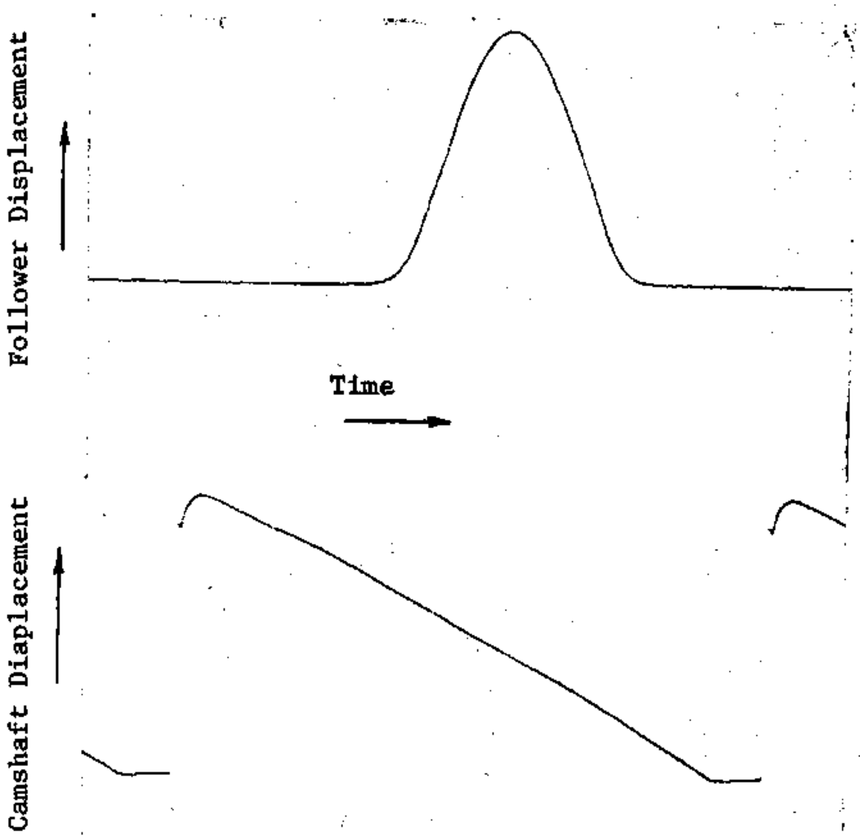


Figure 47. Chart from Sanborn Recorder Showing Follower Displacement and Camshaft Displacement Plotted versus Time

The signal from the rotary potentiometer is fed into one channel of the Sanborn recorder so that the angular camshaft displacement is plotted. The signal from the rotary variable differential transformer is fed into the other channel of the recorder so that the follower displacement is plotted as shown in Figure 47. In this both the camshaft and the follower displacements are plotted against time. The speed of the cam rotation at low speed is not constant because the kinetic energy of the flywheel is drastically reduced, whereas the spring forces either retarding or helping the cam to rotate during a cycle are still of the same magnitude. Therefore, the plot of the camshaft displacement versus time is not linear. To get a true shape of the cam contour, the follower displacement is plotted against the camshaft displacement. This eliminates the parameter time, and eliminates the effect of the fluctuations in speed.

Since the camshaft displacement for one cycle is 360° and the length of one cycle on the output chart is easily observable, this provides all the data necessary to calculate a displacement scale for the potentiometer. To calibrate the rotary differential transformer, the change in displacement that results on the output chart when a feeler gauge is placed between the follower and the cam is recorded. The gain on the recorder's amplifier is then adjusted to give a convenient displacement of the follower as a result of the feeler gauge. This sets the scale for the graph of the follower displacement.

The measurement of the cam acceleration is more difficult. The output signal from the strain gauges on the accelerometer and the signal from the rotary potentiometer are fed into a Sanborn recorder. This output is a plot of the bending moment in the beam of the accelerometer and the cam rotation. The acceleration is proportional to this bending moment and, therefore, the shape of the plot of the acceleration is the same as the shape of the plot of the bending moment. A scale factor for the bending moment measurement is found by calibrating the accelerometer for the bending moment of the beam. The movement on the recorder caused by a known static bending moment in the accelerometer is recorded. Such bending moments are produced by attaching a series of weights at a given location on the beam. In this manner the scale factor for the bending moment in inch-pounds per centimeter of pen displacement on the recorder is determined.

The proportionality factor between the acceleration and the bending moment is found in the manner already described and given by Equation (4-4). The relation between this deflection and the acceleration of the point of attachment to the accelerometer to the follower arm is given by Equation (4-5). Therefore, the expression for the acceleration as a function of the amplitude of the output signal as measured on the recorder is:

$$a(\mathrm{in/sec}^2) = [a/y(1/\mathrm{sec}^2)] \times [y/M_B(1/\mathrm{lb})] \times [M_B/d(\mathrm{in\text{-}lb/cm})] \times d\ (\mathrm{cm})$$

a = Acceleration

y = Deflection of mass on the accelerometer

M_B = Bending moment in accelerometer

d = Output deflection on recorder

Using this, the plot of the bending moment by the recorder can be scaled to read acceleration versus cam rotation.

Experimental Verification of Analytic Procedures

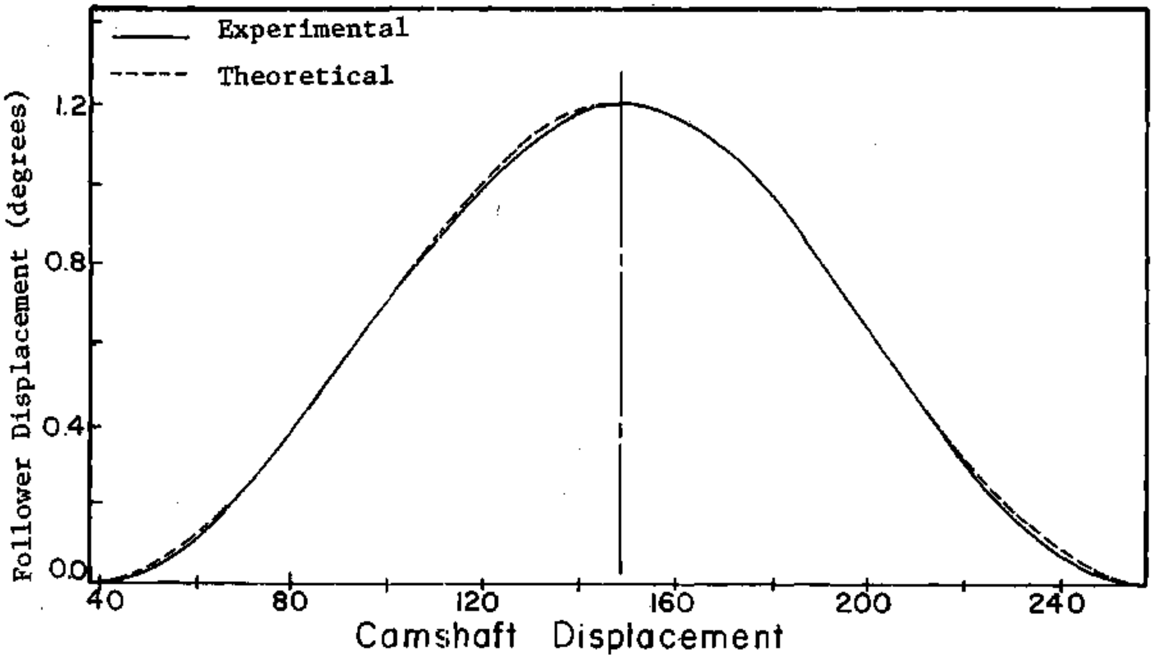
It has been shown that the equations for an elastic curve can be adapted to describe the shape of a cam profile. The characteristics of this cam are further described by velocity and acceleration curves. Then the path of a follower of a given type can be calculated from the cam profile. If this mathematical representation of the cam profile gives the true shape of the cam, then a cam can be made to the given specifications. Therefore, when the displacement and acceleration of a follower operating on this cam are experimentally recorded, they should match the theoretical follower displacement and acceleration curves.

An example elastic band cam is defined by the parameters $\alpha = 110^\circ$, rise = 0.250 inch, base circle radius = 0.500 inch, and a free elastic band. The particular cam with a free elastic band was chosen for the example because the general characteristics of the profile are more dependent on the elastic curve than for other configurations. Even the amount of rise depends upon the elastic curve and not on a fixed locator at the point of maximum rise. An oscillating roller follower, with 3/4 inch roller diameter, on a 12 inch arm is used. Having now defined the cam and follower system, the computer program for the elastic curve configuration (c) is used to obtain a complete description of the cam profile and corresponding follower path. The follower displacement and follower acceleration curves are shown in Figure 48.

To check the validity of these curves, a cam was made to the same parameters that were used to compute the theoretical curves. Then using a follower system of the same dimensions as specified in the theoretical work, the displacement and acceleration of the follower were recorded. Then they were transferred to the charts in Figure 48 so that they could be compared with the predicted curves.

The predicted and recorded paths match very well, except at three points. These disagreements may arise either from manufacturing errors, or from the fact that the measurements were made dynamically on an elastic system, while the theoretical work assumed a cam in static equilibrium.

The record of cam follower displacement can be seen to be slightly asymmetric. This could be due to manufacturing error, or may be an actual dynamic record considering the elastic deflection of the cam. The variations near the point of initial rise and final return are a result of manufacturing problems, due to the very thin section where the



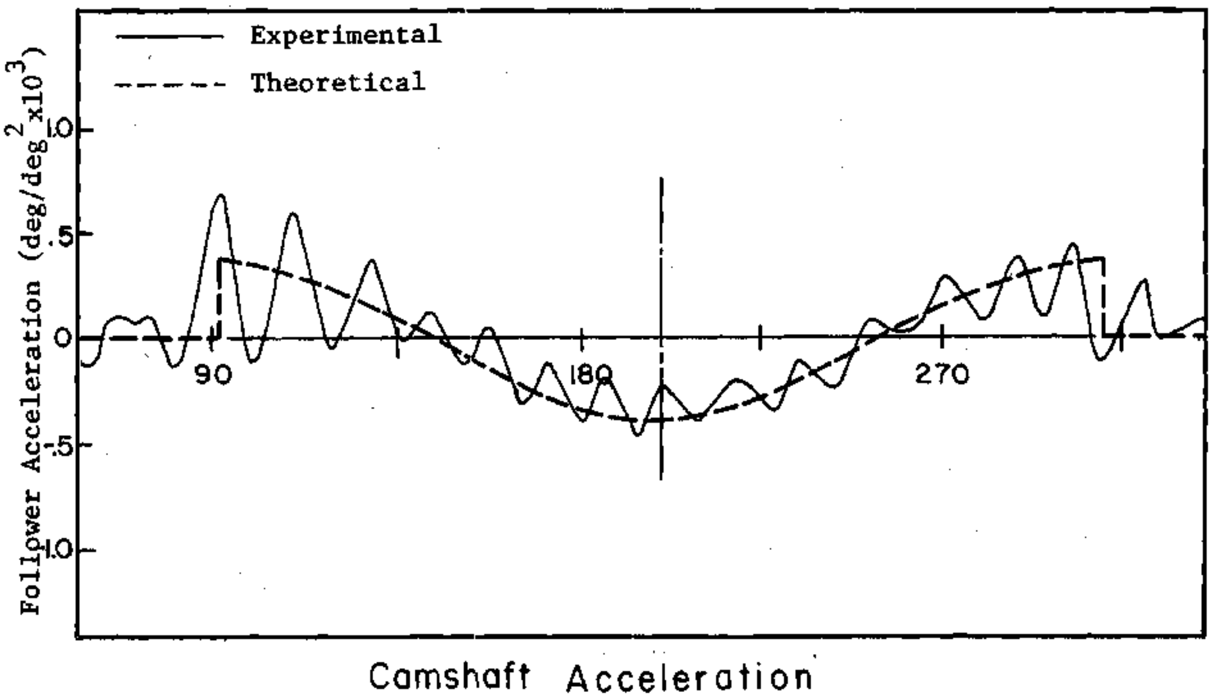


Figure 48. Displacement and Acceleration Curves of Cam with Free Elastic Band, $\alpha = 110°$, $D = 0.75$ inch, and $R = 0.50$ Inch

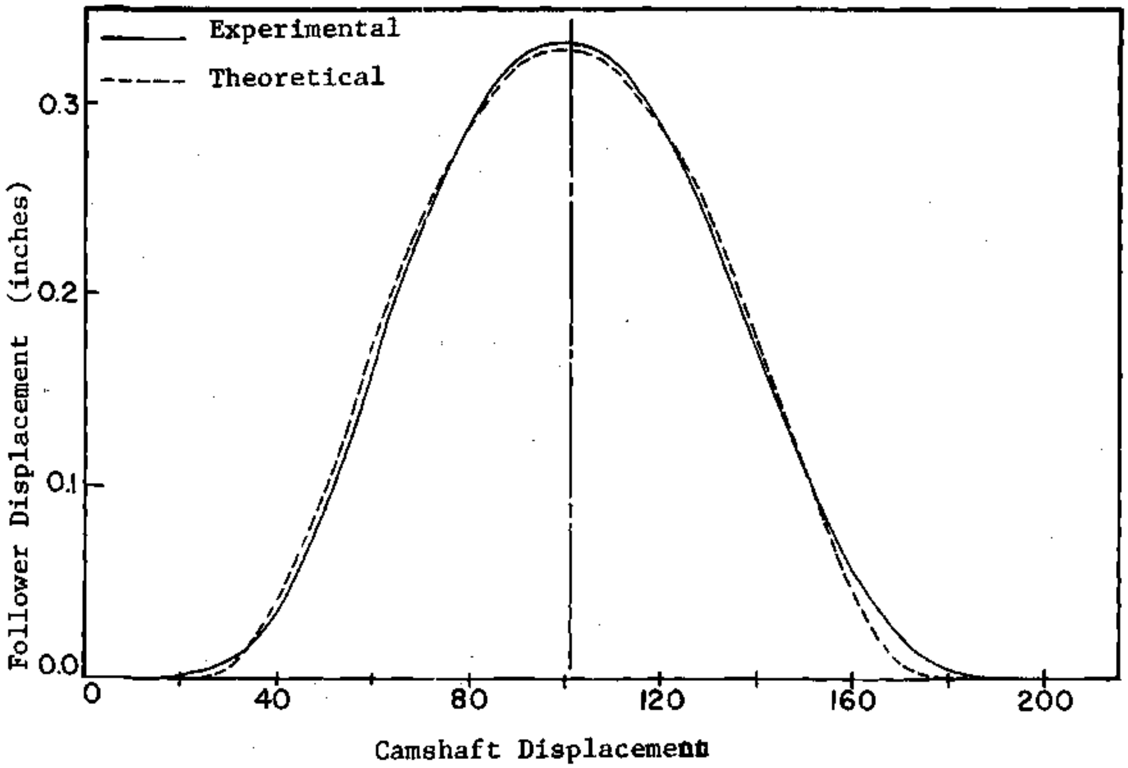
elastic band approaches tangency with the base circle, it is hard to fill this space without pushing the band away from the hub. In the test cam, these spaces appear to have been inadequately filled, and therefore the cam displacement is low.

Comparison between the experimental and calculated acceleration curves also show good agreement. At the points of initial rise and final return, there is a jump in the acceleration, and as expected at such points the actual measured acceleration lags the theoretical acceleration. An abrupt change in acceleration is not physically possible, hence the cam must deform slightly, producing the lag in the measured acceleration. Where there is an abrupt change in theoretical acceleration, the accelerometer will be expected to record an amplitude nearly double the size of the step; and this expectation is fulfilled. The measured values of the negative acceleration on the rise portion of the cam are not as high as predicted. This variation could be the result of the slight imperfection in the cam profile noted in the displacement curve, or due to a deflection of the cam due to the dynamic load.

Two Cams to Give Desired Motion, Matching an Automotive Cam

Two more sample cams were made and tested. In the choice of these, however, there was an additional purpose. Not only could they be used to compare the motions produced with those predicted, as in the case just discussed, but they should be so dimensioned as to produce a desired motion.

It was decided to attempt to match an elastic band cam to an automotive cam. By courtesy of General Motors Corp., the specific rise and angle correlation of an exhaust-valve cam were obtained. In Chapter V, a complete discussion of this design is presented. It is shown that



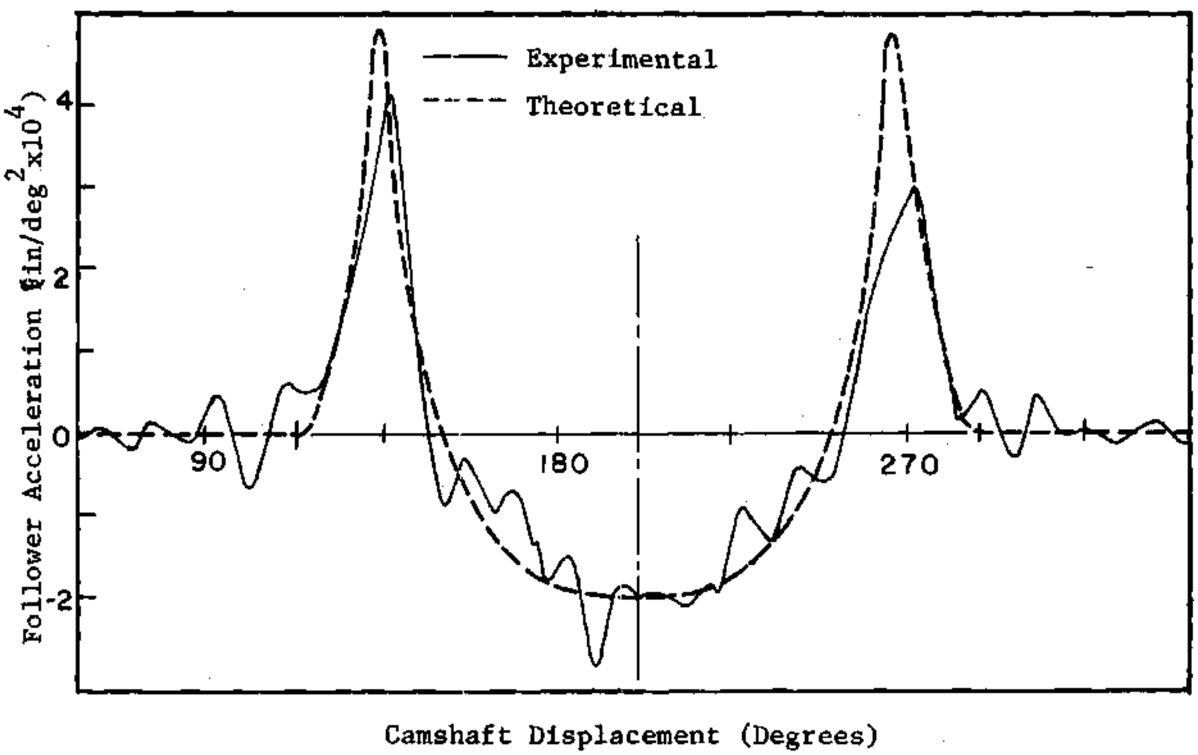


Figure 49. Displacement and Acceleration Curve for Elastic Band Cam: $\alpha = 86°$, $R = 0.50$ Inch, $D = 0.827$ Inch, and $L = 3.615$ Inch

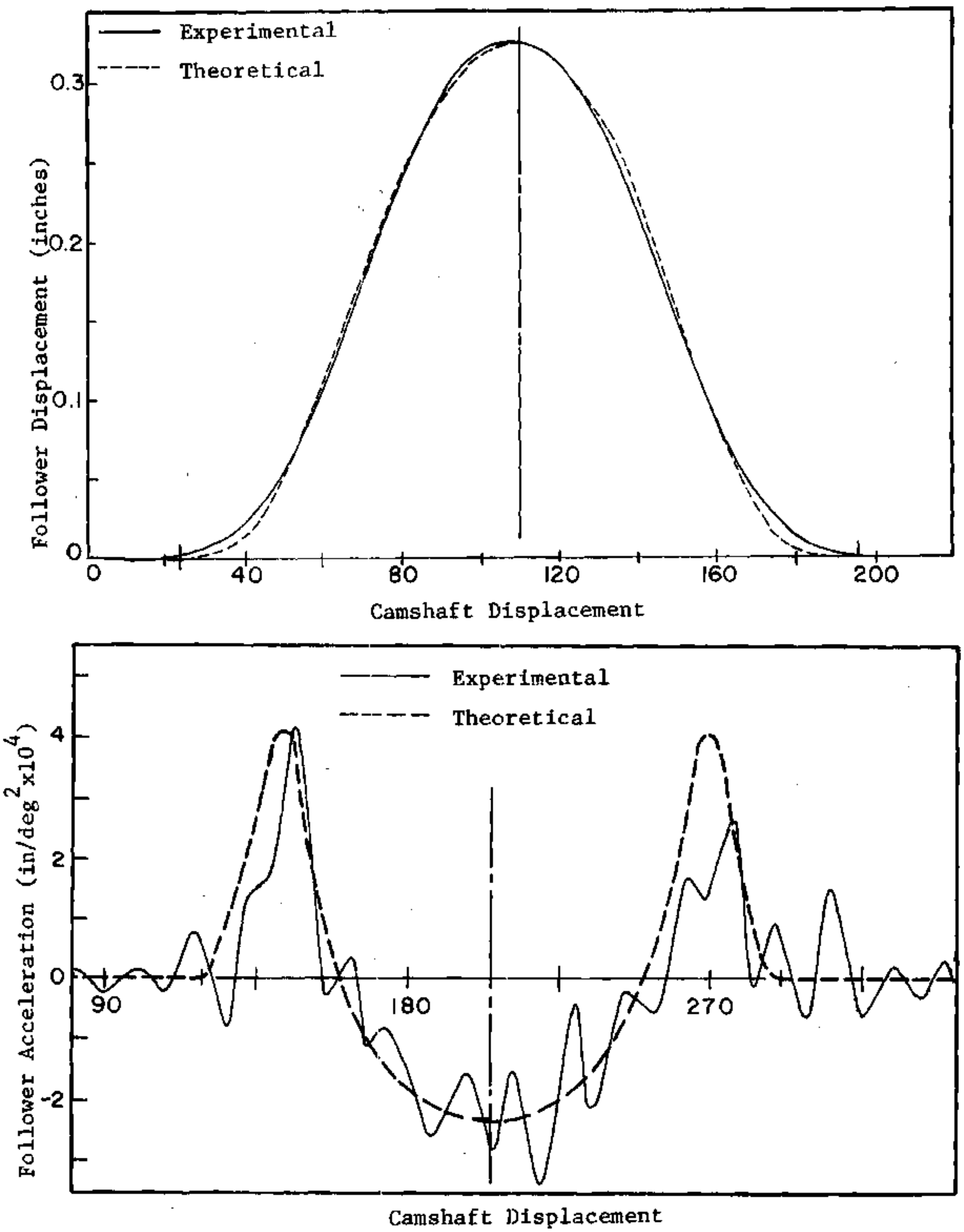


Figure 50. Displacement and Acceleration Curves for Elastic Band Cam: $\alpha = 86°$, $R = 0.654$ Inch, $D = 0.981$ Inch, and $L = 4.571$ Inch

an elastic band with $\alpha = 86°$, R = 0.654 inch, D = 0.981 inch, and L = 4.571 inch is a good match, and so also is an alternative design having $\alpha = 86°$, R = 0.500 inch, D = 0.827, and L = 3.615 inch. The whole angle of rise, and the maximum rise of the two cams are the same. From the computer program for the special case of configuration (a), the graphs of the follower displacement and acceleration curves were obtained and plotted in Figures 49 and 50. The experimentally determined follower displacement and acceleration curves are also included in the charts in these figures for comparison purposes. These examples continue to bear out the good correlation between the analytical and experimental results.

As was found in the first example, there are some slight discrepancies between the experimental and theoretical data. The differences at the ends of the displacement curves are due to the thin section of the matrix. Other deviations are also attributed to manufacturing errors, i.e., the slight error in the location of the toe in Figure 49. The experimental acceleration curves fit the theoretical curves except at the two acceleration peaks during the cycle. The variations at these points can be attributed to the rapid changes in accelerations. The inertia forces on the accelerometer tend to override the vibration forces and produce extended wave lengths of the carrier wave with very high amplitudes. There is a certain amount of time necessary for the accelerometer to respond to the peak accelerations. This delay can be caused by a small deformation of the cam. The acceleration peaks are of a very short duration and by the time the elasticity of the cam restores the deformation, enough of the peak is past that its full height is not recorded.

The tests of elastic band cams are intended to show that the shape of such cams can be predicted using the methods presented in Chapter III. The results of these tests show a definite correlation between the actual and theoretical cams. Some deviations between the two are noticeable, but none would indicate that the actual cam profile was not based on the predicted elastic curve with only minor manufacturing defects.

These three examples in themselves do not prove that the mathematical predictions are true for all configurations, but the mathematics used to predict the shapes of the elastic curves are based on sound principles and would be expected to give correct results. It is sufficient to show that the results can be verified for a few examples. Then it can be concluded that the analytic representations for the other cam configurations are also correct and may be used without further verification.

CHAPTER V

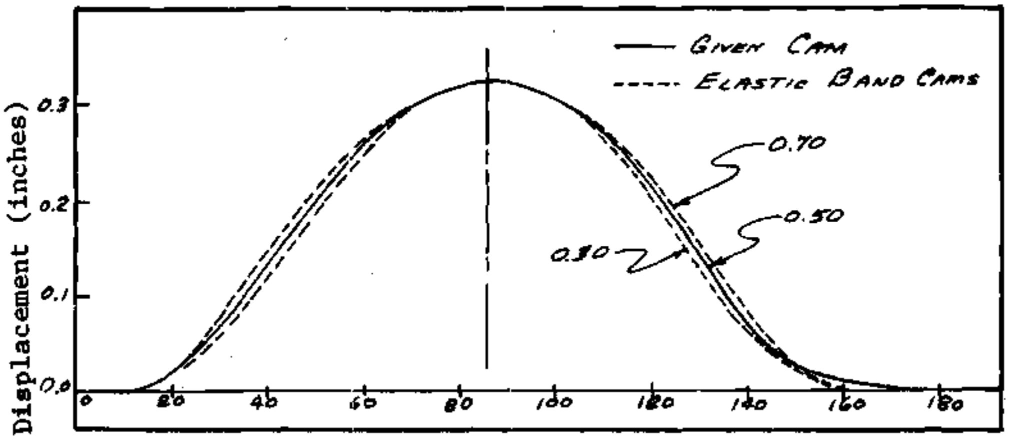
EXAMPLE CAM DESIGN PROBLEM

A method by which an elastic band cam can be designed from the data presented in this dissertation is illustrated by this example. a cam is designed for an exhaust valve cam whose specifications were obtained from Detroit Diesel Engine Division of General Motors Corporation. First, a plot of the displacement and acceleration curves were made from the specifications. The rise portion of this cam covered 86 degrees of camshaft rotation and the return portion covered 117 degrees. However, the cam is almost symmetric about the point of maximum rise. A ramp on the return portion between 160° and 193° differs from the rise portion (Figure 51). Since only symmetric elastic band cams have been investigated and the cam must be designed according to the data presented, and because a ramp cannot be included on an elastic band of this type, a cam with 86 degree rise and 86 degree return is designed. The rise for the elastic band cam is 0.3270 inch as specified. Since the size of the base circle is not stated in the design criterion, different base circle radii, and therefore different rise ratio, are investigated in order to select the best cam of the ones considered.

Given specifications for elastic band cam:

Angle of Rise = 86°

Maximum Rise = 0.3270 inch



Note: Numbers on leaders indicate the rise ratio.

Figure 51. Follower Displacement Curves

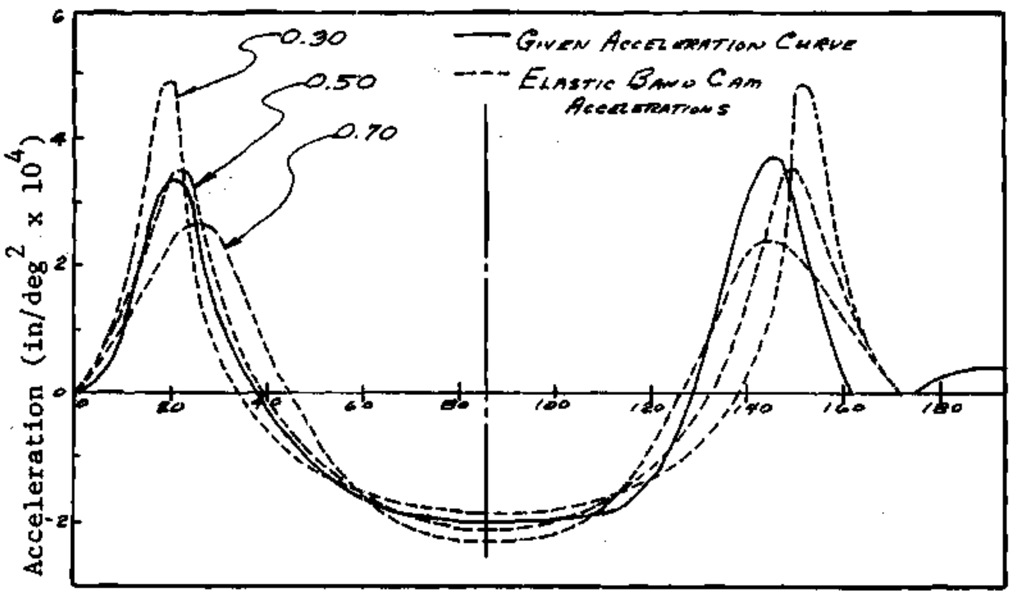


Figure 52. Follower Acceleration Curves

Finite Jerk, i.e. Continuous Acceleration

For rise ratio equal 0.30, the acceleration ratio values for $\alpha = 86°$ are obtained by linear interpolation of values obtained from acceleration characteristics charts for $\alpha = 70°$ and $\alpha = 90°$ (Figure 21).

Initial Acceleration = 0.0

Maximum Positive Acceleration Ratio = $0.000177 \dfrac{1}{\text{degree}^2}$

Maximum Negative Acceleration Ratio = $0.000609 \dfrac{1}{\text{degree}^2}$

Base Circle Radius = $\dfrac{\text{rise}}{\text{rise ratio}}$ = 1.090 inch

Therefore to find actual acceleration:

Maximum Positive Acceleration = (acceleration ratio) x (base circle radius)

= 0.000201 in/deg^2

Maximum Negative Acceleration = 0.000664 in/deg^2

Similar calculations for rise ratio equal 0.50 from Figure 22 yield:

Base Circle Radius = 0.654 inch

Maximum Positive Acceleration = 0.000201 in/deg^2

Maximum Negative Acceleration = 0.000903 in/deg^2

And for rise ratio equal 0.70 from Figure 23 yields:

Base Circle Radius = 0.459 inch

Maximum Positive Acceleration = 0.000199 in/deg^2

Maximum Negative Acceleration = 0.001124 in/deg^2

All of these values of acceleration are for the cam contour, but the important acceleration is that of the follower used with the cam. A translating flat-faced follower is used in this example. To make the necessary changes from the cam acceleration to the follower acceleration, correction factors, the follower acceleration ratio divided by cam acceleration ratio, are calculated from data obtained from the follower output charts for $\alpha = 70^{\circ}$ and $\alpha = 90^{\circ}$ with continuous acceleration (Figures 28 and 29):

$\alpha = 70^{\circ}$			
	Cam	Follower	Correction Factor
Maximum Positive Acceleration Ratio	0.00045	0.00190	4.22
Maximum Negative Acceleration Ratio	0.00250	0.00035	0.14

$\alpha = 90^{\circ}$			
	Cam	Follower	Correction Factor
Maximum Positive Acceleration Ratio	0.00028	0.00052	1.86
Maximum Negative Acceleration Ratio	0.00110	0.00032	0.29

The correction factor for $\alpha = 86°$ is a linear interpolation from these values:

$\alpha = 86°$	
	Correction Factor
Maximum Positive Acceleration Ratio	2.33
Maximum Negative Acceleration Ratio	0.26

Now multiplying the maximum accelerations by these factors, the predicted values of the follower acceleration are found:

Rise Ratio = 0.30

Maximum Positive Acceleration of Follower = 0.000468 in/deg^2

Maximum Negative Acceleration of Follower = 0.000173 in/deg^2

Rise Ratio = 0.50

Maximum Positive Acceleration of Follower = 0.000468 in/deg^2

Maximum Negative Acceleration of Follower = 0.000235 in/deg^2

Rise Ratio = 0.70

Maximum Positive Acceleration of Follower = 0.000464 in/deg^2

Maximum Negative Acceleration of Follower = 0.000292 in/deg^2

A computer program is used to calculate the profiles of the elastic band cams specified by $\alpha = 86°$, rise = 0.327 inch, and rise ratios of 0.30, 0.50, and 0.70. If the computer program had not been available, the calculation of the cam curves could have been carried out by hand. The equations in the section of Chapter III pertaining to the special case of configuration (a) are used. The calculations for rise ratio equal to 0.50 will be outlined. Values R = 0.654 inch, D = 0.981 inch, and $\alpha = 86°$ are substituted into Equations (3-9), (3-12), (3-13), (3-14), and (3-15). A value of ψ_M must be assumed. A first assumption of $\psi_M = 165°$ is used. Calculations give:

$p = 0.9924$

$k = 3.4654$

$\phi_A = 79.707°$

Calculating y_M from Equation (3-5), the radius of curvature of the elastic curve at point A can be found from Equation (3-19).

$r_c = 0.8045$ inch

This value of r_c is greater than the radius of the hub and therefore another assumption for ψ_M must be made. Referring to Figure 10, it can be seen that as the value of ψ_M decreases, the value of the radius of curvature of the elastica decreases, the limiting point occurring at the superimposed ground link, point A. Since the radius of curvature of the elastic curve was greater than required, the new assumption for ψ_M is lower than the original. Continuing this procedure results in a value of ψ_M for which the radius of curvature of the band equals the radius of the hub at the point of contact. For this case:

$\psi_M = 159.8672°$

$p = 0.99914$

$k = 5.56793$

$r_c = 0.5000 = R$

The length of the cam curve as calculated from the Equation (3-18).

$$L' = 0.98866 \text{ inch}$$

The total length of the band is calculated from:

$$L = 2\,L' - 2\pi R\left(\frac{360-2\alpha}{360}\right)$$

$$L = 3.61793 \text{ inches}$$

The shape of the curve can be plotted by calculating coordinate points along the curve using Equations (3-5). It is then transformed to the r,Θ system using Equations (3-27) and (3-28). The velocity and acceleration curves can be obtained from the displacement curve using numerical differentiation.

The follower acceleration curves calculated are shown in Figure 52. The acceleration curve of the Detroit Diesel cam is also included on this chart. All three elastic band cam acceleration curves are similar in shape, but the one for the cam with a rise ratio of 0.50 most closely approximates the desired acceleration curve. Comparing the predicted accelerations with the actual calculated accelerations:

RISE RATIO = 0.30	Predicted (in/deg^2)	Calculated (in/deg^2)
Maximum Positive Acceleration	0.00047	0.00028
Maximum Negative Acceleration	0.00017	0.00025
RISE RATIO = 0.50		
Maximum Positive Acceleration	0.00047	0.00036
Maximum Negative Acceleration	0.00024	0.00022
RISE RATIO = 0.70		
Maximum Positive Acceleration	0.00047	0.00048
Maximum Negative Acceleration	0.00029	0.00019

All of the predictions except for the maximum positive acceleration at rise ratio = 0.30 are close to the calculated values. This shows the use of the charts as a useful tool in the preliminary stages of design of the elastic band cam. The displacement curves for the desired cam and the elastic band cams are shown in Figure 51. Again the cam with rise ratio = 0.50 correlates best.

In this example, the closest correlation of cams as shown in Figure 51 and 52 may not be the best cam for the task. If its base circle is either too large or too small for the application intended, an appropriate base circle must be used and the resulting deviations from the given cam profile considered.

Assuming that the diameter of the base circle of a cam with a rise ratio of 0.50 is acceptable, this cam is chosen as the final design.

Elastic Band Solution:

Rise Ratio - 0.50; Base Circle Radius = 0.654 inch

Using a translating flat-faced follower

Maximum Positive Acceleration = 0.00036 in/deg^2

Maximum Negative Acceleration = 0.00022 in/deg^2

The given Detroit Diesel cam:

Maximum Positive Acceleration = 0.00034 in/deg^2

Maximum Negative Acceleration = 0.00020 in/deg^2

To build a cam to these specifications it is necessary to know the length of the band required as well as the angle of rise, rise, and base circle radius. This length can be estimated from values obtained from Table 3 for α = 70° and 90° and the rise ratio equal 0.50, then interpolating to find the value of the length ratio for a cam with $\alpha = 86°$:

$\alpha = 70°$ Length Ratio = 6.917

$\alpha = 86°$ Length Ratio = 6.993

$\alpha = 90°$ Length Ratio = 7.0092

The actual length of the band is the length ratio multiplied by the base circle radius. Therefore the predicted length of the band required is 4.573 inches. The value of the length calculated by the computer program is 4.571 inches. This is only a 0.002 inch difference in the length of the band between the estimated and calculated lengths. The complete specifications for building the cam are:

Base Circle Radius	= 0.654 inch
Angle of Rise	= 86 degrees
Rise	= 0.327 inch
Length of Band	= 4.571 inches

A cam was built to these specifications to illustrate the feasibility of the construction. Test results for this cam are included in Figure 50 in Chapter IV.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

Evaluation of Elastic Band Cams

The elastic band cam provides solutions to some of the problems of cam design and manufacture. It can be fabricated without using any form of incremental cutting process. The profile which is formed when the elastic band is constrained according to the necessary conditions for a particular cam, will be the exact shape of an elastic curve and not an approximation from a finite number of precision points. Any errors which may occur during the manufacture of the elastic band cam may cause the profile to deviate slightly from the theoretical profile but it will still be an elastic curve. Therefore it will still have continuous values of its derivatives at all points where the theoretical cam profile was continuous. The surface finish on the elastic band cam can be made as smooth as desired by selecting a material for the band which has the desired surface finish. This is an easier method of obtaining a good surface finish than the filing and polishing required by conventional methods.

The analytical descriptions of the follower paths for elastic band cams show that this type cam can be used successfully for many applications. The applications for which it can be used are determined to some extent by the type and dimensions of follower system used. This is because the parameters of the cam specify the actual profile of the cam and not the follower path as is the case for most conventional type cams. Hence the follower path is dependent upon the type of follower system and thus its design is not completely independent. The acceleration characteristics of followers on elastic band cams were compared with the acceleration characteristics of conventional cam profiles but it is hard to draw conclusions from these comparisons without knowing for what application the cam is to be used. The application is of particular importance when comparing elastic band cams because their accelerations are not symmetric with respect to the positive and negative parts. Which of these parts is greater depends upon the type of follower used with the cam. Therefore the application of the cam would determine the merit of a low positive or a low negative acceleration.

One application of cams is in internal combustion engines. The displacement and acceleration curves of an exhaust valve cam used in a diesel engine are used in the example design problem in Chapter V and are shown in Figures 53 and 54. It was found that an elastic band cam with a translating flat-faced follower can be designed to approximate this cam closely. Other very similar curves for cams used in automotive engines were found in the literature (61,62). From these observations it is clear that elastic band cams can be designed which will perform with equal or better results than conventional cams.

We must also consider the economy of manufacture when deciding whether the use of the elastic band cam is merited. From the economy standpoint, the great advantage of the elastic band cam is the low cost of producing the first master cam. This advantage is greatest when the number of cams to be manufactured is small and the percentage of the total cost of each cam from the cost of the master is high. Therefore the savings on the master produce appreciable savings on each cam. For a very high production application, the cost of the original master cam is only a very small part of the total cost of a cam. Therefore the cost saving from using an elastic band cam would not be great enough to merit its use for this reason alone.

The design of the elastic band cam is more complicated than designing a cam to conform to the basic type of conventional profile but with the help of a computer it is no more difficult to design than many numerically based or other specialized cam profiles. Once a computer program is written, the design merely involves substitution in the desired parameters of the rise, angle of rise, base circle radius, and the length of band or acceleration characteristics desired, such as continuous acceleration, and letting the computer turn out a solution of the cam characteristics. These will include the length of band which produces the cam if this is not a parameter. If a computer is not available, the design can be done using a desk calculator and a set of elliptic integral tables.

For a quick design, the data presented in Chapter III can be used to get a good estimate of the length of band necessary to produce a cam conforming to given criterion as was shown in the example in Chapter V.

There are some limitations which result from inherent properties of the elastic band cam. The final cam is made up of more than one material and therefore dependent on the properties of each and the bonding between them. One great limitation of the cam is the lack of load carrying capacity due to the low strength, low elasticity matrix materials. It is found that when high follower loads are on the cam, the bond between the band and matrix breaks. It is unable to withstand the flexing of the band as the two materials deform under load. The higher the follower load, the fewer revolutions required to break this bond. When a spring load varying between 6-1/2 and 8 pounds during the cam rotation is used with a roller follower, 3/4 inch diameter, separation of the band and matrix occurred after about five minutes at 500 revolutions per minute.

For this reason the cam is best used as a master cam. Even for use as a master cam the best procedure is to produce a solid steel master by copying the elastic band cam, then use this steel master as a template to produce all additional cams. Either a hydraulic or electronic servo system or a mechanical system which requires only small copying forces can be used when producing the steel master from the elastic band cam. This two-step procedure overcomes the lack of durability of the elastic band cam. The basic manufacturing benefits of the elastic band cam are preserved because the need for incremental cutting for the fabrication of a master is eliminated and the contour of the cam is actually the mathematical curve that it was designed to be.

The use of an elastic band cam for actual service operation is still a possibility for special cases. The cam can be used advantageously in actual use when there are low follower loads and when the number of cams to be produced is very small. Then their manufacture by assembling them by the elastic band process could be more economical than producing them by copying process. Also to be considered when determining if it is feasible to make cams by the elastic band method is the availability of copying machinery.

The designs that can be accomplished with an elastic band cam are limited. Only symmetric cams can be produced so the design is limited to situations that call for a symmetric cam. This is not too great a handicap because the cams for many situations can be designed symmetrically. The cam profile also cannot be altered to take into consideration deflections in the follower system or peculiar specification of the follower motion such as the ramps which are often used at the beginning or end of the cam rise of an automotive cam. In other words the path of the follower must be accepted as it results from operating on the cam profile which is the elastic curve.

Further Work

There is much that might be done to improve the elastic band cam by eliminating some of the limiting factors. An asymmetric elastic band cam could be developed by using some type of asymmetric loading at the toe of the cam rather than the symmetric load along the cam centerline (Figure 53). This loading would have to be of a type which did not produce any external moments on the band at the point of application. If a moment is produced on the band this will cause a discontinuity in the cam acceleration which is undesirable.

Another similar development would be the adding of an additional constraint along the band to produce modifications in the follower path

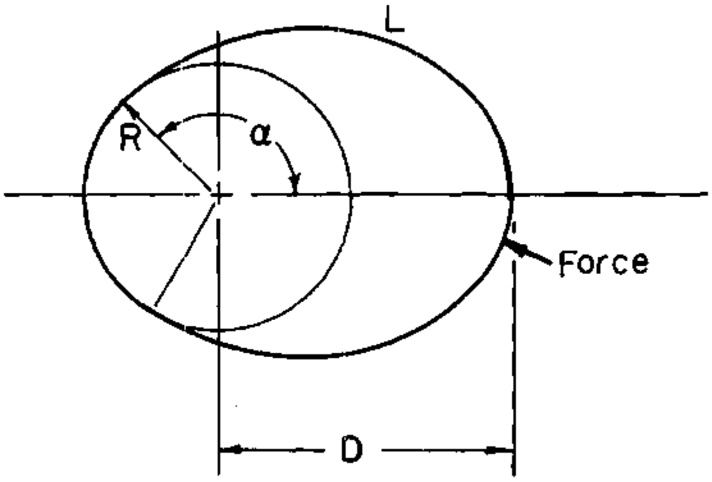


Figure 53. Asymmetric Form of Elastic Band Cam

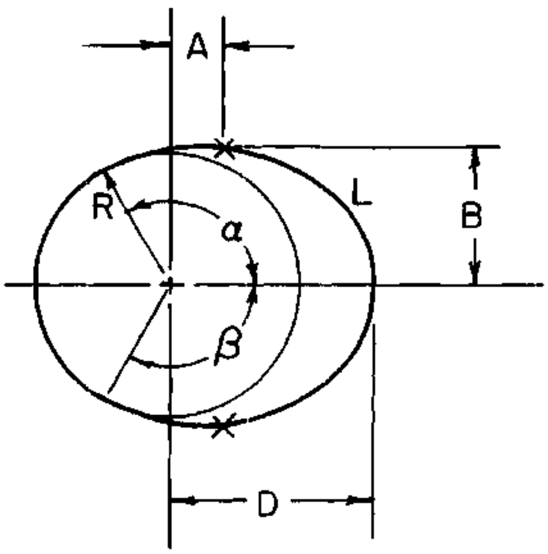


Figure 54. Form of the Elastic Band Cam with Additional Location Constraint of Band

(Figure 54). This could be used to produce ramps at the ends of the rise and return portions of the cam. This is simple in theory but the additional forces on the band would change the mathematic definition of the cam curve.

The calculation of the curves formed for either of the configurations mentioned above could be solved using the principle of elastic similarity, but the procedures would be much more complicated than the ones used in this dissertation. Either the asymmetric or additional force on the band requires that the elastic curve be a portion of two elasticas since the modulus of the elastic curve, p, would be different for the parts of the curve which have different loading. This will increase the number of equations which must be used to describe the fit between the cam curve and the elasticas and therefore increase the difficulty of the solution. The design of cams of these types would be impractical without a computer program to solve the resulting set of equations.

The development of the fabrication of the elastic band cam would probably be the next step in the study of this type cam. Development work with different matrix materials and different methods of joining the ends of the band could help produce a more precise cam. Better casting characteristics for the matrix, such as being able to flow easily into thin sections would be desirable. A precision assembly fixture would be a necessity for the precision production of elastic band cams. Such a fixture which could be used to fabricate all sizes of cams would be desirable. This should incorporate precision methods of setting the angles of rise and amount of rise. Possibly sliding vernier scales attached to swinging clamp used to set the angle of rise and also attached to the locator used to set the amount of rise could be incorporated. This development of the fabrication could also produce improvements which would be beneficial to the production of elastic band cams for production use.

APPENDIX A

DEVELOPMENT OF EQUATIONS FOR ELASTICAS

The development of Equations (3-4) and (3-5), the equations for the undulating and the nodal elasticas, respectively, are presented here as taken from *Flexible Bars* by R. Frish-Fay. These equations define the elasticas by defining the basic strut sections which are used to form the elasticas.

The Undulating Elastica

The undulating elastica is formed from a series of columns deflected by a vertical force, as described in Chapter III, and shown in Figures 2 and 55. This column configuration is called a basic strut. The column is fixed at the bottom and subject to a vertical load P at the top. If the bar is sufficiently flexible, it will take the shape of Figure 55. The flexural rigidity EI is constant, a condition that will hold throughout this discussion. The bending moment at B(x,y) due to P is $M = -Py = EI/r$, and

$$y = -\frac{EI}{PR} = -\frac{1}{k^2 r} \qquad \text{(A-1)}$$

where $k = (P/EI)^{\frac{1}{2}}$ and r is the radius of curvature.
Using the accurate expression of 1/r we have:

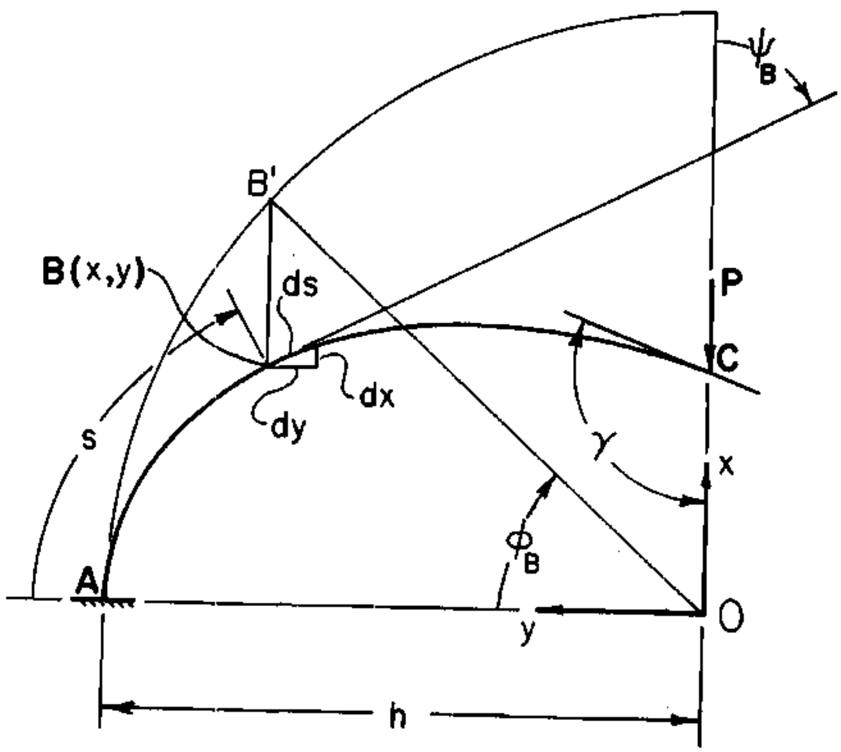


Figure 55. Basic Strut Configuration for Undulating Elastica

$$y = -\frac{d^2y/dx^2}{k^2[1 + (dy/dx)^2]^{1/2}} \tag{A-2}$$

Integrating with respect to x, Equation (A-2) becomes:

$$y^2 = \frac{2}{k^2[1 + (dy/dx)]^{1/2}} + C \tag{A-3}$$

From Figure 55, $dx/ds = \cos\psi$, hence $[1 + (dy/dx)^2]^{1/2} = 1/\cos\psi$ and Equation (A-3) reduces to:

$$y^2 = \frac{2}{k^2} \cos\psi + C = \frac{2}{k^2} [1 - 2 \sin^2(\psi/2)] + C \tag{A-4}$$

Introducing:

$$h^2 = 2/k^2 + C \tag{A-5}$$

we get:

$$y^2 = h^2 - \frac{4}{k^2} \sin^2(\psi/2) \tag{A-6}$$

At the fixed end A, $\psi = 0$, hence $y_A = h = AO$. Since y^2 is positive, the solution of Equation (A-6) will depend on the relative values of h and k.

First we assume: $h^2 < 4/k^2$

Let

$$h^2 = 4p^2/k^2 \tag{A-7}$$

where $p < 1$ and select ϕ such that:

$$\sin(\psi/2) = p \sin\phi \tag{A-8}$$

In this case

$$y = h \cos\phi \tag{A-9}$$

Noting that $dy/d\phi = -h \sin\phi$ and $dy/ds = \sin\psi$, after some reductions:

$$ds = -\frac{h\, d\phi}{2p(1 - p^2\sin^2\phi)^{1/2}} = -\frac{d\phi}{k(1 - p^2\sin^2\phi)^{1/2}} \tag{A-10}$$

ψ is negative as shown in Figure 55, hence ϕ is also negative. The negative sign in Equation (A-10) means that ϕ is decreasing while s increases. Integrating ds from 0 to s and disregarding the negative sign:

$$s = \frac{1}{k}\int_0^{\phi} \frac{d\phi}{(1 - p^2\sin^2\phi)^{1/2}} = \frac{1}{k} F(p,\phi) \tag{A-11}$$

This equation contains the unknowns p and ϕ. The modulus p will be calculated from the assumption that the length of the strut does not alter during bending. Hence:

$$L = \frac{1}{k} \int_0^{\pi/2} \frac{d\phi}{(1 - p^2 \sin^2\phi)^{1/2}} = \frac{1}{k} K(p) \tag{A-12}$$

the limits having been chosen such that ϕ sweeps the quadrant in Figure 55 from AO to C'O. Since, from (A-8):

$$p = \frac{\sin(\psi/2)}{\sin\phi}$$

the slope at C is found from:

$$\sin(\gamma/2) = p \tag{A-13}$$

The total horizontal deflection of the bar is:

$$AO = h = 2p/k$$

and the y coordinate of B:

$$y = \frac{2p \cos\phi}{k} \tag{A-14}$$

Turning now to the vertical coordinate of B we note that:

$$dx = ds\ \cos\psi = ds(1 - 2p^2\sin^2\phi) \tag{A-15}$$

Substituting ds from expression (A-10) in Equation (A-14):

$$dx = \frac{(1 - 2p^2\sin^2\phi)d\phi}{k(1 - p^2\sin^2\phi)^{1/2}}$$

and

$$x = \frac{1}{k}\int_0^{\phi} \frac{d\phi}{(1 - p^2\sin^2\phi)^{1/2}} - \frac{2p^2}{k}\int_0^{\phi} \frac{\sin^2\phi\ d\phi}{(1 - p^2\sin^2\phi)^{1/2}}$$

The second integral equals:

$$[F(p,\phi) - E(p,\phi)]/p$$

and we find for the vertical deflection at B:

$$x = 2E(p,\phi)/k - F(p,\phi)/k \tag{A-16}$$

The Equations (A-14) and (A-16) are the equations for the x,y coordinates of the undulating elastica and Equation (A-11) is the equation for the length of an arc on this elastica. These are the Equations (3-4) given in Chapter III.

The Nodal Elastica

The nodal elastica is formed from a series of columns deflected by a vertical force and a moment as shown in Figures 5 and 56. The column is defined exactly as the column for the undulating elastica. Referring back to the development of the equations for the basic strut forming the undulating elastica, if in Equation (A-6) $h \leq 2/k$, y will vanish for certain values of x (for $h = 2/k$, y vanishes at $x = \pm\infty$). Since from $y = -1/k^2r$, the curvature vanishes with y, it is clear that points of contraflexure occur whenever the curve passes through the x-axis. If, on the other hand, $h > 2/k$, y and the curvature will never vanish; in other words, there will be no points of contraflexure.

Let $\quad 4/k^2 = h^2p^2, \quad (p^2 < 1)$

Then, from Equation (A-6):

$$y = h[1 - p^2\sin^2(\psi/2)]^{1/2} \tag{A-17}$$

Since $p^2 < 1$, y will not vanish for any real value of ψ. We now select a parameter ϕ such that:

$$\sin\phi = p\,\sin(\psi/2) \tag{A-18}$$

It follows at once that:

$$y = h\,\cos\phi = \frac{2}{pk}\cos\phi \tag{A-19}$$

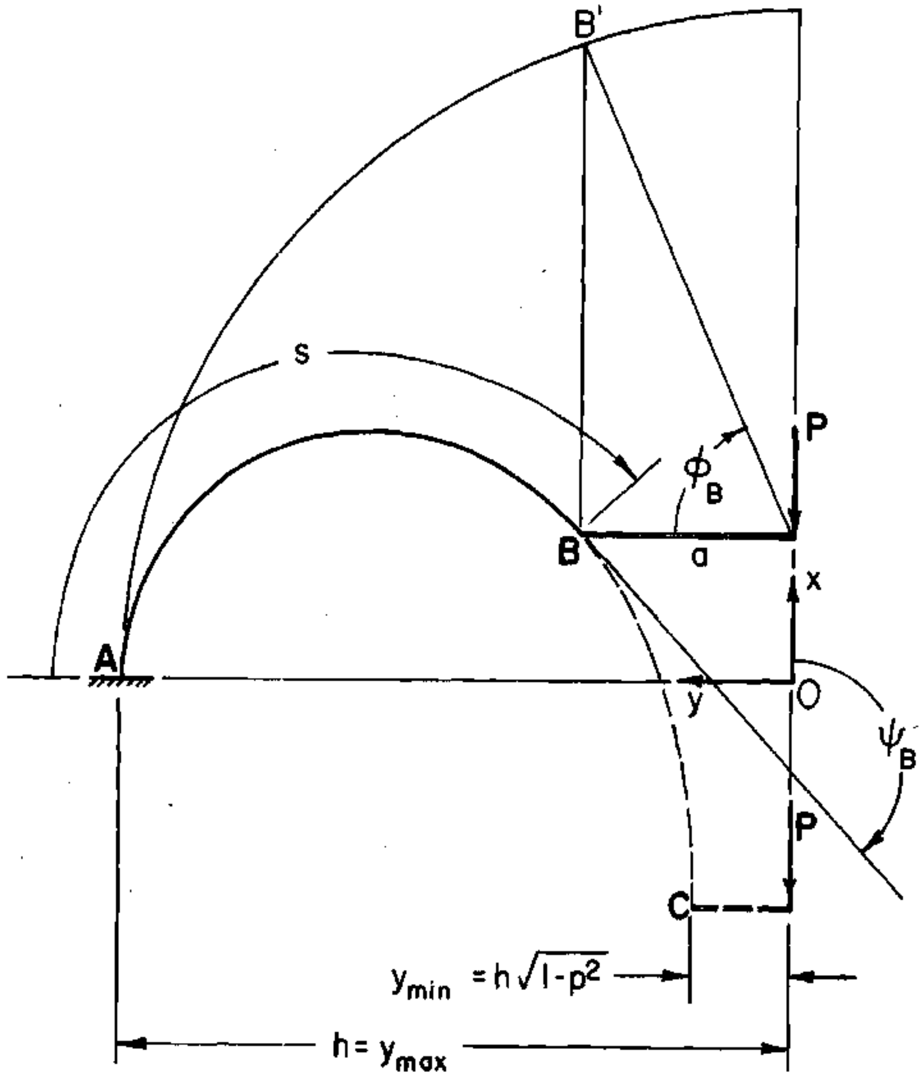


Figure 56. Basic Strut Configuration for Nodal Elastica

Also, from:

$$\frac{dy}{d\psi} = - \frac{hp^2 \sin\psi}{4[1 - p^2 \sin^2(\psi/2)]^{1/2}}$$

and from:

$$\sin\psi = \frac{dy}{d\psi} \cdot \frac{d\psi}{ds}$$

we obtain:

$$ds = - \frac{hp^2 d(\psi/2)}{2[1 - p^2 \sin^2(\psi/2)]^{1/2}}$$

and:

$$s = - \frac{hp^2}{2} \int_0^{\psi/2} \frac{d(\psi/2)}{[1 - p^2 \sin^2(\psi/2)]^{1/2}} \tag{A-20}$$

$$= - hp^2 \; F(p,\psi/2)/2$$

The reason for the negative sign is that ψ is decreasing from the fixed end to the free end. In the following discussion s is taken positive.

To find x we note that:

$$dx = \cos\psi \; ds = [1 - 2 \sin^2(\psi/2)]ds$$

$$= \frac{hp^2 d(\psi/2)}{2[1 - p^2 \sin^2(\psi/2)]^{1/2}} - \frac{hp^2 \sin^2(\psi/2) \; d(\psi/2)}{[1 - p^2 \sin^2(\psi/2)]^{1/2}}$$

(the negative sign in ds has been neglected). Integrating:

$$x = hp^2F(p,\psi/2)/2 - hp^2 \int_0^{\psi} \frac{\sin^2 (\psi/2)\ d(\psi/2)}{[1 - p^2\sin^2(\psi/2)]^{1/2}} \tag{A-21}$$

$$= h\ E(p,\psi/2) - h(1 - \frac{p^2}{2})\ F(p,\psi/2)$$

It follows from Equation (A-17) that $y_{max} = h$ and $y_{min} = h(1 - p)^{1/2}$. This latter value of y occurs when $\psi = \pi$. Since y_{max} and y_{min} have the same sign, y cannot be zero: consequently P cannot act directly on the bar, because at P, y = 0. Therefore, the load can act only through a rigid lever which is equivalent to a couple Ph $\cos\phi$ and a direct load P. This is shown in Figure 56. The elastic shape of this curve is called the *nodal elastica*. The Equations (A-19) and (A-21) are the equations for the x,y coordinates of the nodal elastica and Equation (A-20) is the equation for the length of an arc on this elastica. These are the Equations (3-5) given in Chapter III.

APPENDIX B

SAMPLE COMPUTER PROGRAMS

The computer programs for the seven cam curve configurations, (a) through (g), and the special case for a cam with a continuous acceleration are included in this Appendix. The programs were written in Algol-60 and run on a Burroughs B-5500 computer at the Rich Electronic Computer Center at Georgia Tech. Figure 58 is the flow diagram of these programs.

There are parts of each of these programs which are the same for all configurations. Such parts are only included once in this listing. These are the List and Format statements, the elliptic integral procedures, calculation of the shape of the cam curve the follower path calculations, the numerical differentiation, and the harmonic analysis. The section in which the cam curve is described by calculating a series of coordinate points along the cam curve and then transforming them to the (r,Θ) coordinate system, is the same for all configurations except for minor variations. These exceptions are noted in this section of the program. The procedures used to evaluate the elliptic integrals are based on a method presented by L. V. King (62). The harmonic analysis was based on the numerical method of finding the Fourier coefficients presented by Sokolnikoff and Redheffer (63). In the procedures for calculating the follower paths the dimensions of the follower systems are specified by the variables:

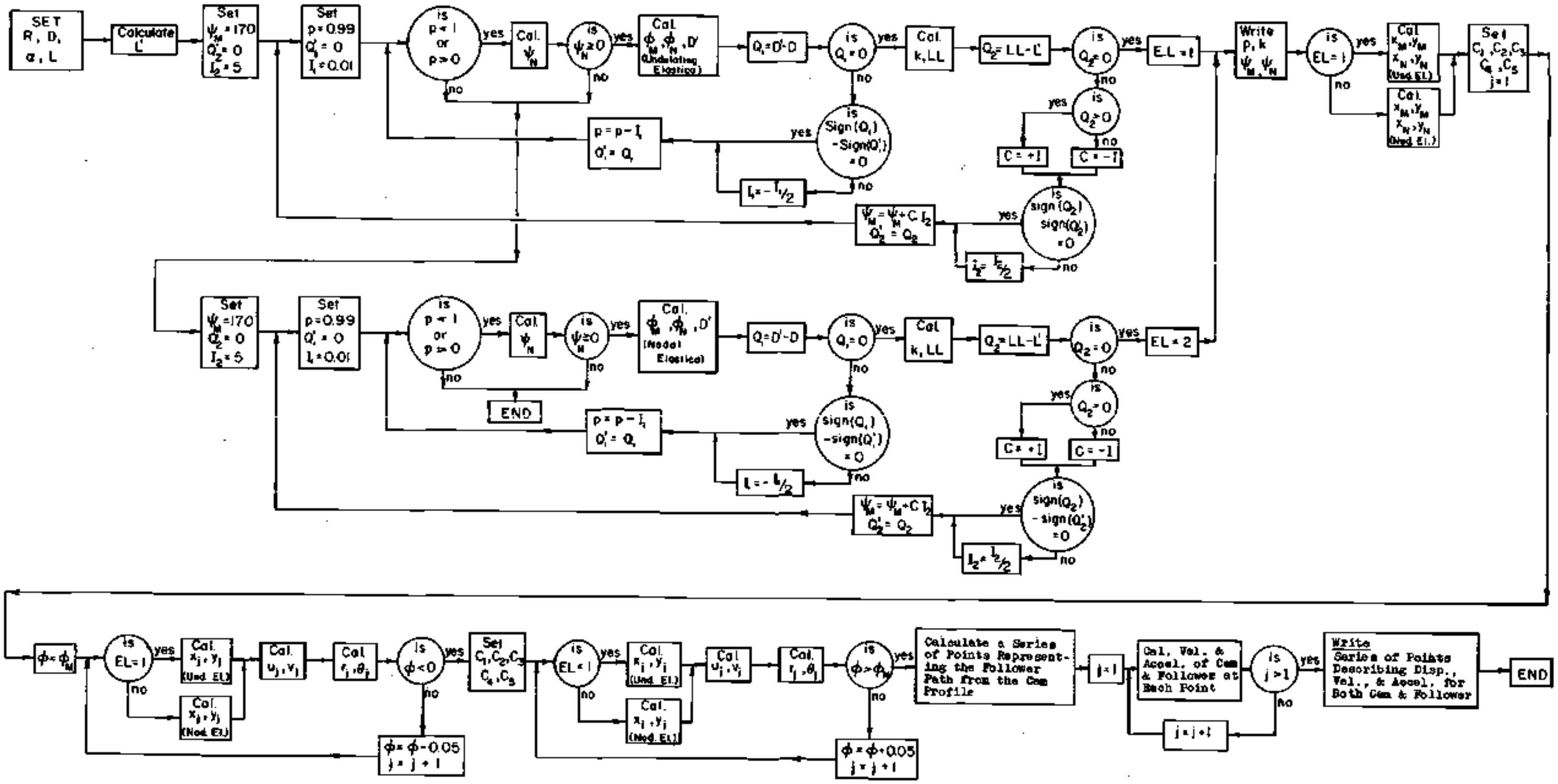


Figure 57. Flow Diagram for Computer Programs

RR = roller radius or the distance from the face to the arm centerline,

AR = the offset of a translating follower or the vertical distance from the follower pivot center to the camshaft center,

BR = the horizontal distance from the follower pivot to the camshaft, and

CR = the length of the follower arm.

A program segment for the determination of the fit between the cam curve and an elastica is given for each configuration. These are the solutions for the systems of equations derived in Chapter III. The results from these segments are the values of p, k, ϕ_M, and ϕ_N.

PROGRAMS FOR AN ELASTIC BAND CAM TO FIND: (1) THE VALUES OF THE JACOBIAN ELLIPTIC INTEGRALS, (2) THE VALUES OF THE PARAMETERS WHICH DEFINE THE FIT BETWEEN THE CAM CURVE AND AN ELASTICA, (3) A SERIES OF COORDINATE POINTS ALONG THE CAM CURVE WHICH WHEN TRANSFORMED TO THE (R,THETA) SYSTEM DEFINE THE CAM PROFILE, (4) THE PATHS OF FOLLOWERS OPERATING ON THE CAM, (5) THE VELOCITY AND ACCELERATION OF THE CAM AND FOLLOWER AND, (6) THE CONSTITUENT HARMONICS OF THE FOLLOWER ACCELERATION CURVE

* *

PROCEDURES FOR EVALUATING THE JACOBIAN ELLIPTIC INTEGRALS OF THE FIRST AND SECOND KIND BOTH COMPLETE AND INCOMPLETE

ELLIPTIC INTEGRAL, FIRST KIND, INCOMPLETE

```
REAL PROCEDURE F(K,PHIR);VALUE K,PHIR;REAL K,PHIR;BEGIN INTEGER I,N,J;RE00000100
AL KPIM,FJ,DEL,TEMPC,TEMPS,PHM,FI,AM;ARRAY A,B,PH[0:2];LABEL L1;FORMAT T00000200
L("FOR STATEMENT  COMPLETE, IF STATEMENT NOT SATISFIED");J←1;I←2;KPIM←SQ00000300
RT(1.0-(K×K));FJ←0.0;A[J]←1.0;B[J]←KPIM;PH[J]←PHIR;FOR N←1STEP 1UNTIL 1000000400
0DD BEGIN FJ←FI;A[I]←(1.0/2.0)×(A[I-1]+B[I-1]);B[I]←SQRT(A[I-1]×B[I-1]);00000500
TEMPC←COS(PH[J]);TEMPS←SIN(PH[J]);DEL←SQRT(A[J]×A[J]×TEMPC×TEMPC+B[J]×B[00000600
J]×TEMPS×TEMPS);PH[I]←ARCTAN(TAN(PH[J])×SQRT(A[I]/A[J])×SQRT((B[J]+DEL)/00000700
(A[J]+DEL)));FI←PH[I]/A[I];A[J]←A[I];B[J]←B[I];PH[J]←PH[I];IF ABS(FI-FJ)00000800
<1.0@-11THEN GO TO L1;END;WRITE(PRINT,TL);L1:F←FI;END;                  00000900
```

ELLIPTIC INTEGRAL, FIRST KIND, COMPLETE

```
REAL PROCEDURE KC(K);VALUE K;REAL K;BEGIN REAL KPIM,AM;INTEGER I,N,J;ARR00001000
AY A,B[0:2];LABEL L1;FORMAT TL("FOR STATEMENT  COMPLETE, IF STATEMENT NO00001100
T SATISFIED");J←1;I←2;KPIM←SQRT(1.0-(K×K));A[J]←1.0;B[J]←KPIM;FOR N←1STE00001200
P 1UNTIL 10000 BEGIN A[I]←(0.5)×(A[I-1]+B[I-1]);B[I]←SQRT(A[I-1]×B[I-1])00001300
;IF ABS(A[I]-B[I])<1.0@-11THEN GO TO L1;A[J]←A[I];B[J]←B[I];END;WRITE(PR00001400
INT,TL);L1:AM←(1.5707963268)/A[I];KC←AM;END;                            00001500
```

ELLIPTIC INTEGRAL, SECOND KIND, INCOMPLETE

```
REAL PROCEDURE E(K,PHIR);VALUE K,PHIR;REAL K,PHIR;BEGIN INTEGER I,N,J,M;00000100
```

```
REAL G,CSTN1,CSTN2,EC1,EC2,KPIM,Y,Z,PI;REAL ARRAY A,B,C,PH[0:2];LABEL L1
,L2;FORMAT TL("FOR STATEMENT  COMPLETE, IF STATEMENT NOT SATISFIED");PI←
(3.1415926536);G←2.0×PI;J←1;I←2;EC1←EC2←0.0;KPIM←SQRT(1.0-(K×K));A[J]←1.
0;B[J]←KPIM;C[J]←K;CSTN1←0.0;CSTN2←C[J]×C[J];PH[J]←PHIR;FOR N←1STEP 1UNT
IL 100DO BEGIN EC2←EC1;A[I]←(0.5)×(A[J]+B[J]);B[I]←SQRT(A[J]×B[J]);C[I]←
(0.5)×(A[J]-B[J]);M←-1;Z←PH[J];L2:Z←Z-G;M←M+1;IF Z>0.0THEN GO TO L2;Z←Z+
G;Y←ARCTAN((B[J]/A[J])×TAN(PH[J]));IF(Z>(PI/2.0))AND(Z<((3.0×PI)/2.0))TH
EN Y←Y+PI;IF Y<0.0THEN Y←Y+G;PH[I]←PH[J]+(Y+G×M);CSTN1←CSTN1+C[I]×SIN(PH
[I]);CSTN2←CSTN2+((2.0*N)×C[I]×C[I]);EC1←CSTN1+(1.0-(0.5)×CSTN2)×(PH[I]/
((2.0*N)×A[I]));IF(ABS(EC2-EC1)<1.0@-11)AND(ABS(A[I]-B[I])<1.0@-11)THEN
GO TO L1;A[J]←A[I];B[J]←B[I];C[J]←C[I];PH[J]←PH[I];END;WRITE(PRINT,TL);L
1:E←EC1;END;
```

ELLIPTIC INTEGRAL, SECOND KIND, COMPLETE

```
REAL PROCEDURE EC(K);VALUE K;REAL K;BEGIN INTEGER I,N,J;REAL KPIM,EC1,EC
2,CSTN;REAL ARRAY A,B,C[0:2];LABEL L1;FORMAT TL("FOR STATEMENT  COMPLETE
, IF STATEMENT NOT SATISFIED");J←1;I←2;EC1←EC2←0.0;KPIM←SQRT(1.0-(K×K));
A[J]←1.0;B[J]←KPIM;C[J]←K;CSTN←C[J]×C[J];FOR N←1STEP 1UNTIL 100DO BEGIN
EC2←EC1;A[I]←(0.5)×(A[J]+B[J]);B[I]←SQRT(A[J]×B[J]);C[I]←(0.5)×(A[J]-B[J
]);CSTN←CSTN+C[I]×C[I]×(2.0*N);EC1←(1.0-0.5×(CSTN))×(1.5707963268/(A[I])
);IF(ABS(EC2-EC1)<1.0@-11)AND(ABS(A[I]-B[I])<1.0@-11)THEN GO TO L1;A[J]←
A[I];B[J]←B[I];C[J]←C[I];END;WRITE(PRINT,TL);L1:EC←EC1;END;
```

* *

LIST AND FORMAT STATEMENTS USED FOR ALL PROGRAMS

```
LIST AN2(R,D,APA,THA) ;
LIST AN3(P,K,L,PHA,PHB,PHC,L1 ) ;
LIST AN6(MPA,MNA,MJK) ;
LIST AN10( MAH     ,MAC,MJC ) ;
LIST AN11(S,A[0],FOR J←1 STEP 1 UNTIL 10 DO [J, A[J] ],
        FOR J←1 STEP 1 UNTIL 10 DO [J, B[J] ]) ;
LIST AN12(RR,AR,BR,CR) ;
LIST AN13( TH[N],RS[N],VE,AC,TPS,X[N],TPC,VEL,ACC ) ;
LIST AN99(TA,CC) ;
```

```
FORMAT TL01(X5,"BASED UPON THE UNDULATING ELASTICA ", //),
       TL02(X5,"BASED UPON THE NODAL ELASTICA ", //) ,
       TL2(X5,"CONFIGURATION  (D)   ",///,
         X5,  "GIVEN PARAMETERS: ",///,
         X10,"R =",F8.4,X7," D =",F8.4,X4,"ALPHA =",F9.4,X4,"THETA A =",
         F9.4,X4,//),
      TL3( X5,"CALCULATED VALUES OF ELASTIC CURVE ",///,
         X10,"P  =",F11.5,X4,"K  =",F11.5,X4,"L  =",F10.5,// ,
         X10,"PH[A] =",F10.4,X5,"PH[B] =",F10.4,X5,"PH[C] =",F10.4,// ,
         X10,"TOTAL LENGTH OF BAND =",F8.5,///),
       TL6(X5,"MAX POSITIVE ACCEL =",R16.8,X5,"MAX NEGATIVE ACCEL ="
         R16.8, //,X5,"MAXIMUM JERK = ",R16.8,/ ),
       TL10(X15,"COMPARISON VALUES ",X5,"FOR SIMPLE HARMONIC MOTION",///,
         X10,"MAX ACCEL =",F9.5,X5,"MAX JERK =  INFINITE ",///,
         X5,"FOR CYCLOIDAL MOTION",//,
         X10,"MAX ACCEL =",F9.5,X5,"MAX JERK =",F9.5,//),
       TL11(//,"HARMONIC ANALYSIS OF CAM ACCEL ",//,X14,"N",X15,"N=1",/,
         X5,"Y = A0 + S  AK COS(K×X) + S  BK SIN(K×X)",
         X20,"N =",I3,/,X13,"K=1",X14,
         "K=1",//,X4,"COEFFICIENTS:",///,X9,"A[0]=",F10.5,///,
         2(X9,5 ("A[",I1,"]=",F10.5,X4),//),//,2( X9,5("B[",I1,"]=",
         F10.5,X4 ),//),X5,"CHECK",///) ,
       TL12(X10,"RR =",F8.4,X3,"AR =",F8.4,X3,"BR =",F8.4,X3,"CR =",F8.4
        ,///,X13,"ACTUAL CAM CONTOUR",X38,"PATH OF FOLLOWER CENTER",///,
         X2,"THETA",X9,"R",X10,"VEL",X9,"ACCEL",X12 ,
       X2,"THETA",X9,"R",X8 ,"PHI ",X5,"ACCEL R",X5,"ACCEL PHI",X12,//),
       TL13( 2(F9.5,X2),2(F10.5,X2),X9,3(F9.5,X2),2(F10.5,X2)),
       TL21("FOLLOWER MOTION OF ROLLER FOLLOWER ON RADIAL ARM ",///),
       TL220("FOLLOWER MOTION OF RECIPROCATING ROLLER FOLLOWER",///),
       TL23("FOLLOWER MOTION OF FLAT FACED FOLLOWER ON RADIAL ARM",///),
       TL24("FOLLOWER MOTION OF RECIPROCATING FLAT FACED FOLLOWER",///),
       TL99(      X10,"THETA = ",R14.6,X5,"R = ",R14.6, ),
       NOTE1("P NOT IN RANGE FOR UNDULATED ELASTICA, TRY NODUAL ",
             "ELASTICA" ///) ,
       NOTE2("NO REAL SOLUTION FOR P ",///) ,
       NOTE3("N > 100 , OUT ",///) ,
       NOTE4("REAL SOLUTION FOR ELASTIC CURVE BUT PHYSICALLY IMPOSSIBLE"
```

```
    ," TO CONSTRUCT CAM " , //);
```

* *

SEGMENTS OF THE PROGRAMS TO EVALUATE THE PARAMETERS P, K, PSI M, AND PSI N, TO DEFINE THE ELASTICA AND WHAT PORTION OF IT IS FITTED BY THE CAM CURVE

```
        CONSTANTS:
        PI←(3.1415926536)    ;
        G ← PI/180.0 ;
 IF THE FIT IS WITH THE UNDULATING ELASTICA, EL = 1,
 IF THE FIT IS WITH THE NODAL ELASTICA, EL = 2.

%                                   CONFIGURATION (A)

     P = P, K = K, PSI M = THA, PSI N = THC
        R ← 0.5 ;
        FOR APA ← 110.0 DO
        FOR D ← 0.75  DO
        FOR L1 ← 3.600 DO
BEGIN   APAR ← APA × G ;
        WRITE(PRINT [PAGE] ) ;
        WRITE(PRINT,TL2,AN2) ;
% SOLVE FOR  P AND K BY TRIAL AND ERROR
%CALCULATIONS FOR THE NODAL ELASTICA
        LL ← (L1 - 2×R ×(PI - APAR))/ 2.0 ;
        THA ← 170.0 ;    THAR ← THA × G ;
        QPM2 ← 0.0 ;     IT2 ← 5.0 × G ;
LB8:
        P ← 0.99999999 ;
        IT1 ← 0.02 ;
        M ← 0 ;
        N1 ← 0 ;
        QPM1 ← 0.0 ;
LB2:
        IF (P>1) OR (P<0.0)  THEN
```

```
BEGIN      WRITE(PRINT,NOTE2) ;
           GO TO JOB ;                                                    END ;
           THBR ← PI ;
           THCR ← 2×PI − APAR − THAR ;
           IF THCR < 0.0 THEN GO TO JOB ;
           TP1 ← P×SIN(THAR/2) ;
           PHAR ← ARCTAN(TP1/ SQRT(1−TP1*2)) ;
           TP1 ← P×SIN(THCR/2) ;
           PHCR ← ARCTAN(TP1/ SQRT(1−TP1*2)) ;
           C1 ← THAR/2 ;
           C2 ← THCR/2 ;
           TPS ← COS(PHCR) − COS(PHAR) ;
           TPC ← TPS/(E(P,C2) +E(P,C1) −2×EC(P) −(1−P*2/2)×
                 (F(P,C2) +F(P,C1) −2×KC(P))) ;
           IF ABS(TPC ) < 1.0@−10  THEN
LB22:      IF M = 0 THEN
BEGIN      N1 ← 0 ;
           IT1 ← 0.02 ;
           QPM1 ← 0.0 ;
           P ← P − 0.010 ;
           M ← 1 ;   GO TO LB2 ;                                        END
           ELSE
BEGIN      WRITE(PRINT,NOTE2) ;     GO TO JOB ;                         END ;
           TA ← COS(THCR)/ TPC −SIN(THCR) ;
           IF ABS(TA) < 1.0@−10  THEN  GO TO LB22 ;
           DD← R×(COS(THAR)/ TPC +SIN(THAR))/ (COS(THCR)/ TPC −SIN(THCR));
           Q1 ← DD − D ;
IF ABS(Q1)>1.0@−6THEN BEGIN IF N1<100THEN BEGIN IF(QPM1≠0.0)AND(SIGN(Q1)
−SIGN(QPM1)≠0.0)THEN IT1←−IT1/2.0;P←P−IT1;IF P<0.2THEN BEGIN WRITE(PRINT
,NOTE1);GO TO JOB;END;QPM1←Q1;N1←N1+1;GO TO LB2;END ELSE BEGIN WRITE(PRI
NT,NOTE3);GO TO JOB;END;END;
           K ← (2/P) ×TPS/ (D×COS(THCR) −R×COS(THAR)) ;
           H ← 2/(P×K) ;
           L ← (P/K) ×(2×KC(P) −F(P,C1) −F(P,C2)) ;
           Q2 ← L − LL ;
IF ABS(Q2)>0.000001THEN BEGIN IF N2<100THEN BEGIN IF(Q2>0)THEN CJ←−1ELSE
 CJ←1;IF(QPM2≠0)AND((SIGN(Q2)−SIGN(QPM2))≠0.0)THEN IT2←IT2/2;THAR←THAR+C
```

```
J×IT2;THA←THAR/G;QPM2←Q2;N2←N2+1;GO TO LB8;END;END;WRITE(PRINT[PAGE]);WR00013500
ITE(PRINT,TL2,AN2);
          L1 ← 2×L +2×R×(PI -APAR) ;                                       00013600
          EL ← 2 ;
          PHBR← ARCTAN(P  / SQRT(1 -P*2)) ;                                    0680
          PHA ← PHAR / G ;
          PHB ← PHBR / G ;                                                     0740
          PHC ← PHCR / G ;                                                     0750
LB4:                                                                           0760
          WRITE(PRINT,TL3,AN3) ;                                               0800
          IF EL = 1 THEN
          WRITE(PRINT,TL01)                                                    0830
          ELSE                                                                 0840
          WRITE(PRINT,TL02) ;                                                  0850
JOB:                                                                           0860
                    CONFIGURATION (A)   SPECIAL CASE, CONT. ACCEL.

     P = P, K = K, PSI M = THA, PSI N = THC
          R ← 0.50 ;
          FOR APA ← 110.0 DO
          FOR D ← 0.75 DO
BEGIN     APAR ← APA × G ;
          WRITE(PRINT [PAGE] ) ;
          WRITE(PRINT,TL2,AN2) ;
% SOLVE FOR  P AND K BY TRIAL AND ERROR
%CALCULATIONS FOR THE NODAL ELASTICA
          THA ← 170.0 ;   THAR ← THA × G ;
          IT2 ← 0.5 × G ;
          QPM2 ← 0.0 ;
LB8:      P ← 0.99999999 ;
          IT1 ← 0.02 ;
          M ← 0 ;
          N1 ← 0 ;
          QPM1 ← 0.0 ;
LB2:
          IF (P>1) OR (P<0.0)  THEN
BEGIN     WRITE(PRINT,NOTE2) ;
```

```
        GO TO JOB ;                                                     END ;
        THBR ← PI ;
        THCR ← 2×PI - APAR - THAR ;
        IF THCR < 0.0 THEN GO TO JOB ;
        TP1 ← P×SIN(THAR/2) ;
        PHAR ← ARCTAN(TP1/ SQRT(1-TP1*2)) ;
        TP1 ← P×SIN(THCR/2) ;
        PHCR ← ARCTAN(TP1/ SQRT(1-TP1*2)) ;
        C1 ← THAR/2 ;
        C2 ← THCR/2 ;
        TPS ← COS(PHCR) - COS(PHAR) ;
        TPC ← TPS/(E(P,C2) +E(P,C1) -2×EC(P) -(1-P*2/2)×
             (F(P,C2) +F(P,C1) -2×KC(P))) ;
        IF ABS(TPC ) < 1.0@-10  THEN
LB22:   IF M = 0 THEN
BEGIN   N1 ← 0 ;
        IT1 ← 0.02 ;
        QPM1 ← 0.0 ;
        P ← P - 0.010 ;
        M ← 1 ;    GO TO LB2 ;                                        END
        ELSE
BEGIN   WRITE(PRINT,NOTE2) ;     GO TO JOB ;                          END ;
        TA ← COS(THCR)/ TPC -SIN(THCR) ;
        IF ABS(TA) < 1.0@-10  THEN  GO TO LB22 ;
        DD← R×(COS(THAR)/ TPC +SIN(THAR))/ (COS(THCR)/ TPC -SIN(THCR));
        Q1 ← DD - D ;
        IF ABS(Q1) > 1.0@-5   THEN
BEGIN   IF N1 < 100 THEN
BEGIN   IF (QPM1 ≠ 0.0) AND (SIGN(Q1) - SIGN(QPM1) ≠ 0.0) THEN
        IT1 ← -IT1 / 2.0 ;
        P ← P - IT1 ;
        QPM1 ← Q1 ;
        N1 ← N1 + 1 ;
        GO TO LB2 ;                                                   END
        ELSE
BEGIN   WRITE(PRINT, NOTE3) ;
      GO TO JOB ;                                                     END ; END ;
```

```
          K ← (2/P) ×TPS/ (D×COS(THCR) −R×COS(THAR)) ;
          H ← 2/(P×K) ;
          L ← (P/K) ×(2×KC(P) −F(P,C1) −F(P,C2)) ;
          L1 ← 2×L +2×R×(PI −APAR) ;
          EL ← 2 ;
          YA ← H × COS(PHAR) ;
          Q2 ← 1 / (K*2 × YA ) −R ;
          IF ABS(Q2) > 0.0001 THEN
BEGIN     IF N2 < 100 THEN
BEGIN     IF(Q2 <0) THEN CJ ← 1 ELSE CJ ← −1 ;
          IF (QPM2≠0) AND((SIGN(Q2)−SIGN(QPM2)) ≠ 0.0 ) THEN
          IT2 ← IT2/2 ;
          THAR ← THAR +  CJ ×IT2 ;
          THA ← THAR / G ;
          QPM2 ← Q2 ;
          N2 ← N2 + 1 ;
          GO TO LB8 ;                                 END ; END ;
          WRITE(PRINT,TL2,AN2) ;
          PHBR← ARCTAN(P  / SQRT(1 −P*2)) ;
          PHA ← PHAR / G ;
          PHB ← PHBR / G ;
          PHC ← PHCR / G ;
LB4:
          WRITE(PRINT,TL3,AN3) ;
          IF EL = 1 THEN
          WRITE(PRINT,TL01)
          ELSE
          WRITE(PRINT,TL02) ;
JOB:

%                           CONFIGURATION (B)

     P = P, K = K, PSI M = THA, PSI N = THB
          R ←0.5 ;
          FOR APA ← 110.0 DO
          FOR D ← 0.75 DO
          FOR L1 ← 3.640 DO
```

```
BEGIN    APAR ← APA × G ;
         WRITE(PRINT [PAGE]) ;
         WRITE(PRINT,TL2,AN2) ;
% SOLVE FOR  P AND K BY TRIAL AND ERROR
%CALCULATIONS FOR THE UNDULATING ELASTICA
         LL ← (L1 - 2×R ×(PI - APAR))/ 2.0 ;
         THA ←  APA+10 ; THAR ← THA × G ;
         QPM2 ← 0.0 ;     IT2 ← 5.0 × G ;
LB8:
         P ← 0.99999999 ;
         IT1 ← 0.02 ;
         M ← 0 ;
         N1 ← 0 ;
         QPM1 ← 0.0 ;
LB1:
         IF (P>1) OR (P<0.0)  THEN
BEGIN    WRITE(PRINT,NOTE1) ;
         GO TO LB3 ;                                    END ;
         THBR ← THAR - APAR ;
         TPS ← SIN(THAR/2) ;
         IF TPS > P THEN
BEGIN    WRITE(PRINT,NOTE1) ;
         GO TO LB3 ;                                    END ;
         PHAR ← ARCTAN(TPS/ SQRT(P*2 -TPS*2)) ;
         TPS ← SIN(THBR/2) ;
         PHBR ← ARCTAN(TPS/ SQRT(P*2 -TPS*2)) ;
         TP1 ← APAR + THBR ;
         TPS ←2×P×( COS(PHBR) - COS(PHAR)) ;
         TPC ← TPS/ (2×E(P,PHAR) -2×E(P,PHBR) -F(P,PHAR) +F(P,PHBR)) ;
         IF ABS(TPC ) < 1.0@-10  THEN
LB11:    IF M = 0 THEN
BEGIN    N1 ← 0 ;
         IT1 ← 0.02 ;
         QPM1 ← 0.0 ;
         P ← P - 0.010 ;
         M ← 1 ;   GO TO LB1 ;                          END
         ELSE
```

```
BEGIN    WRITE(PRINT,NOTE2) ;      GO TO JOB ;                  END ;
         TA ← COS(THBR)/TPC +SIN(THBR) ;
         IF ABS(TA  ) < 1.0@-10  THEN
         GO TO LB11 ;
         DD← (R×(COS(TP1)/TPC +SIN(TP1)))/( COS(THBR)/TPC +SIN(THBR )) ;
         Q1 ← DD - D ;
IF ABS(Q1)>1.0@-6THEN BEGIN IF N1<100THEN BEGIN IF(QPM1≠0.0)AND(SIGN(Q1)00012700
-SIGN(QPM1)≠0.0)THEN IT1←-IT1/2.0;P←P-IT1;IF P<0.2THEN BEGIN WRITE(PRINT00012800
,NOTE1);GO TO JOB;END;QPM1←Q1;N1←N1+1;GO TO LB2;END ELSE BEGIN WRITE(PRI00012900
NT,NOTE3);GO TO JOB;END;END;
                                                                    00013000
         K ←   TPS/(-R×COS(TP1) +D×COS(THBR)) ;
         EL ← 1 ;
                                                                        0680
         H ← 2×P/K ;
         L ← (F(P,PHAR) - F(P,PHBR))/ K ;
         Q2 ← L - LL ;
IF ABS(Q2)>0.000001THEN BEGIN IF N2<100THEN BEGIN IF(Q2>0)THEN CJ←-1ELSE00013300
 CJ←1;IF(QPM2≠0)AND((SIGN(Q2)-SIGN(QPM2))≠0.0)THEN IT2←IT2/2;THAR←THAR+C00013400
J×IT2;THA←THAR/G;QPM2←Q2;N2←N2+1;GO TO LB8;END;END;WRITE(PRINT[PAGE]);WR00013500
ITE(PRINT,TL2,AN2);
                                                                    00013600
         L1 ← 2×L + 2×R×(PI-APAR) ;
         GO TO LB4 ;
LB3:
%CALCULATIONS FOR NODAL ELASTIC
         LL ← (L1 - 2×R ×(PI - APAR))/ 2.0 ;
         THA ←  APA+10 ; THAR ← THA × G ;
         QPM2 ← 0.0 ;    IT2 ← 5.0 × G ;
LB9:
         P ← 0.99999999 ;
         IT1 ← 0.02 ;
         M ← 0 ;
         N1 ← 0 ;
         QPM1 ← 0.0 ;
LB2:
         IF (P>1) OR (P<0.0)  THEN
BEGIN    WRITE(PRINT,NOTE2) ;
         GO TO JOB ;                                          END ;
         THBR ← THAR - APAR ;
```

```
          TP1 ← P×SIN(THAR/2) ;
          PHAR ← ARCTAN(TP1/ SQRT(1 -TP1*2)) ;
          TP1 ← P×SIN(THBR/2) ;
          PHBR ← ARCTAN(TP1/ SQRT(1 -TP1*2)) ;
          TP1 ← APAR + THBR ;
          C1 ← THAR/2 ;
          C2 ← THBR/2 ;
          TPS ← COS(PHBR) - COS(PHAR) ;
          TPC ← TPS/ (E(P,C1) -E(P,C2) -(1-P*2/2)×(F(P,C1) -F(P,C2))) ;
          IF ABS(TPC ) < 1.0@-10  THEN
LB22:     IF M = 0 THEN
BEGIN     N1 ← 0 ;
          IT1 ← 0.02 ;
          QPM1 ← 0.0 ;
          P ← P - 0.010 ;
          M ← 1 ;    GO TO LB2 ;                              END
          ELSE
BEGIN     WRITE(PRINT,NOTE2) ;      GO TO JOB ;               END ;
          TA ← COS(THBR)/ TPC + SIN(THBR) ;
          IF ABS(TA   ) < 1.0@-10  THEN
          GO TO LB22 ;
          DD← (R×(COS(TP1)/TPC +SIN(TP1)))/( COS(THBR)/TPC +SIN(THBR )) ;
          Q1 ← DD - D ;
IF ABS(Q1)>1.0@-6THEN BEGIN IF N1<100THEN BEGIN IF(QPM1≠0.0)AND(SIGN(Q1)
-SIGN(QPM1)≠0.0)THEN IT1←-IT1/2.0;P←P-IT1;IF P<0.2THEN BEGIN WRITE(PRINT
,NOTE1);GO TO JOB;END;QPM1←Q1;N1←N1+1;GO TO LB2;END ELSE BEGIN WRITE(PRI
NT,NOTE3);GO TO JOB;END;END;
          K ← (2/P)×TPS/(-R×COS(TP1) +D×COS(THBR)) ;
          EL ← 2 ;
          H ← 2/(P×K) ;
          L ← (P/K)×(F(P,C1) - F(P,C2)) ;
          Q2 ← L - LL ;
IF ABS(Q2)>0.000001THEN BEGIN IF N2<100THEN BEGIN IF(Q2>0)THEN CJ←-1ELSE
 CJ←1;IF(QPM2≠0)AND((SIGN(Q2)-SIGN(QPM2))≠0.0)THEN IT2←IT2/2;THAR←THAR+C
J×IT2;THA←THAR/G;QPM2←Q2;N2←N2+1;GO TO LB8;END;END;WRITE(PRINT[PAGE]);WR
ITE(PRINT,TL2,AN2));
          L1 ← 2×L + 2×R×(PI-APAR) ;
```

```
LB4:
          PHA ← PHAR / G ;
          PHB ← PHBR / G ;
          WRITE(PRINT,TL3,AN3) ;
          IF EL = 1 THEN
          WRITE(PRINT,TL01)
          ELSE
          WRITE(PRINT,TL02) ;
J08:

%                                         CONFIGURATION (C)

     P = P, K = K, PSI M = THA, PSI N = THB
          R ←0.5 ;
          FOR D ← 0.75 DO
          FOR APA ← 110.0 DO
BEGIN     APAR ← APA × G ;
% SOLVE FOR  P AND K BY TRIAL AND ERROR
%CALCULATIONS FOR THE UNDULATING ELASTICA
          EL ← 1 ;
          WRITE(PRINT [PAGE] ) ;
          WRITE(PRINT,TL01)    ;
          WRITE(PRINT,TL2,AN2) ;
          P ← 0.9999999999    ;
          IT ← 0.02 ;
          M ← 0 ;
          N ← 0 ;
          QPM ← 0.0 ;
          TPS ← SIN(APAR) ;
          TPC ← COS(APAR) ;
          TP1 ← D − R×COS(APAR) ;
          C1 ← SIN(APAR/2) ;
LB1:
          IF (P >1.0) OR (P < 0.0)  THEN
BEGIN     WRITE(PRINT,NOTE1) ;       GO TO LB3 ;              END ;
          IF C1 > P THEN
          IF M = 0 THEN
```

```
BEGIN     PHBR ← PI/2 ;
          P ← C1 ;
          M ← 1 ;                                              END
          ELSE
BEGIN     WRITE(PRINT,NOTE1) ;
          GO TO LB3 ;                                          END
          ELSE
          PHBR ← ARCTAN(C1/ SQRT(P*2 - C1*2)) ;
         K ← 2×P×(1 - COS(PHBR))/ TP1 ;
          Q ← ((2 ×E(P,PHBR) - F(P,PHBR))/ K - (R×TPS));
          IF ABS(Q ) > 1.0@-5  THEN
BEGIN     IF N < 100 THEN
BEGIN     IF (QPM ≠ 0.0 ) AND (SIGN(Q)-SIGN(QPM) ≠ 0.0) THEN
          IT ← -IT / 2.0 ;
          P ← P - IT ;
          QPM ← Q ;
          N ← N + 1 ;
          GO TO LB1 ;                                          END
          ELSE
BEGIN     WRITE(PRINT,NOTE1) ;
          GO TO LB3 ;                                          END ; END ;
          H ← (2.0×P ) / K ;
          L ← F(P,PHBR) / K ;
          L1 ← 2×L +2×R×(PI -APAR) ;
          GO TO LB4 ;
LB3:
%CALCUALTIONS FOR NODAL ELASTICA
          EL ← 2 ;
          WRITE(PRINT [PAGE] ) ;
          WRITE(PRINT,TL02) ;
          WRITE(PRINT,TL2,AN2) ;
          P ← 0.999999 ;
          IT ← 0.02 ;
          QPM ← 0.0 ;
          N ← 0 ;
          C1 ← APAR/2 ;
LB2:
```

```
            IF (P >1.0) OR (P < 0.0)   THEN
BEGIN       WRITE(PRINT,NOTE2) ;       GO TO JOB ;               END ;
            K ← (2/(P×R×SIN(APAR))) ×(E(P,C1) -(1-P↑2/2)× F(P,C1)) ;
            IF ABS(K)  < 1.0@-9   THEN
BEGIN       P ← P - IT ;       GO TO LB2 ;                       END ;
            H ← 2 / (P×K) ;
            Q ← H×(1-SQRT(1-P↑2×SIN(C1)↑2)) -D +R×COS(APAR) ;
            IF ABS(Q ) > 1.0@-5  THEN
BEGIN       IF N < 100 THEN
BEGIN       IF (QPM ≠ 0.0 ) AND (SIGN(Q)-SIGN(QPM) ≠ 0.0) THEN
            IT ← -IT / 2.0 ;
            P ← P - IT ;
            QPM ← Q ;
            N ← N + 1 ;
            GO TO LB2 ;                                          END
            ELSE
BEGIN       WRITE(PRINT,NOTE2) ;
            GO TO JOB ;                                          END ; END ;
            TP1 ← P×SIN(C1) ;
            PHBR ← ARCTAN(TP1/ SQRT(1 - TP1↑2)) ;
            L ← (P/K) ×F(P,C1) ;
            L1 ← 2×L +2×R×(PI -APAR) ;
LB4:
            PHA ← 0.0 ;
            PHB ← PHBR / G ;
            PHC ← 0.0 ;
            WRITE(PRINT,TL3,AN3) ;
JOB:

%                                 CONFIGURATION (D)

      P = P, K = K, PSI M = THA, PSI N = THC
            R ←0.5 ;
            FOR APA ← 110.0 DO
            FOR D ← 0.75 DO
            FOR L1 ← 3.670 DO
BEGIN       APAR ← APA × G ;
```

```
            WRITE(PRINT [PAGE]) ;
            WRITE(PRINT,TL2,AN2) ;
% SOLVE FOR  P AND K BY TRIAL AND ERROR
%CALCULATIONS FOR THE UNDULATING ELASTICA
            LL ← (L1 - 2×R ×(PI - APAR))/ 2.0 ;
            THA ← 170.0 ;    THAR ← THA × G ;
            QPM2 ← 0.0 ;     IT2 ← 5.0 × G ;
LB8:
            P ← 0.99999999 ;
            IT1 ← 0.02 ;
            M ← 0 ;
            N1 ← 0 ;
            QPM1 ← 0.0 ;
LB1:
            IF (P>1) OR (P<0.0)  THEN
BEGIN       WRITE(PRINT,NOTE1) ;
            GO TO LB3 ;                                  END ;
            THBR ← APAR - THAR ;
            TPS ← SIN(THAR/2) ;
            IF TPS > P THEN
BEGIN       WRITE(PRINT,NOTE1) ;
            GO TO LB3 ;                                  END ;
            PHAR ← ARCTAN(TPS/ SQRT(P*2 -TPS*2)) ;
            TPS ← SIN(THBR/2) ;
            IF TPS > P THEN
BEGIN       WRITE(PRINT,NOTE1) ;
            GO TO LB3 ;                                  END ;
            PHBR ← ARCTAN(TPS/ SQRT(P*2 -TPS*2)) ;
            TP1 ← THAR ;
            TPS ←2×P×( COS(PHBR) - COS(PHAR)) ;
            TPC ← TPS/ (2×E(P,PHAR) +2×E(P,PHBR) -F(P,PHAR) -F(P,PHBR)) ;
            IF ABS(TPC ) < 1.0@-10  THEN
LB11:       IF M = 0 THEN
BEGIN       N1 ← 0 ;
            IT1 ← 0.02 ;
            QPM1 ← 0.0 ;
            P ← P - 0.010 ;
```

```
          M ← 1 ;   GO TO LB1 ;                                    END
          ELSE
BEGIN     WRITE(PRINT,NOTE2) ;      GO TO JOB ;                    END ;
          TA ← COS(THBR)/TPC -SIN(THBR) ;
          IF ABS(TA  ) < 1.0@-10  THEN
          GO TO LB11 ;
          DD← (R×(COS(TP1)/TPC +SIN(TP1)))/( COS(THBR)/TPC -SIN(THBR )) ;
          Q1 ← DD - D ;
IF ABS(Q1)>1.0@-6THEN BEGIN IF N1<100THEN BEGIN IF(QPM1≠0.0)AND(SIGN(Q1)
-SIGN(QPM1)≠0.0)THEN IT1←-IT1/2.0;P←P-IT1;IF P<0.2THEN BEGIN WRITE(PRINT
,NOTE1);GO TO JOB;END;QPM1←Q1;N1←N1+1;GO TO LB2;END ELSE BEGIN WRITE(PRI
NT,NOTE3);GO TO JOB;END;END;
          K ←  TPS/(-R×COS(TP1) +D×COS(THBR)) ;
          EL ← 1 ;
          H ← 2×P/K ;
          L ← (F(P,PHAR) + F(P,PHBR))/ K ;
          Q2 ← L - LL ;
IF ABS(Q2)>0.000001THEN BEGIN IF N2<100THEN BEGIN IF(Q2>0)THEN CJ←-1ELSE
 CJ←1;IF(QPM2≠0)AND((SIGN(Q2)-SIGN(QPM2))≠0.0)THEN IT2←IT2/2;THAR←THAR+C
J×IT2;THA←THAR/G;QPM2←Q2;N2←N2+1;GO TO LB8;END;END;WRITE(PRINT[PAGE]);WR
ITE(PRINT,TL2,AN2);
          L1 ← 2×L + 2×R×(PI-APAR) ;
          GO TO LB4 ;
LB3:
%CALCULATIONS FOR NODAL ELASTIC
          LL ← (L1 - 2×R ×(PI - APAR))/ 2.0 ;
          THA ← 170.0 ;   THAR ← THA × G ;
          QPM2 ← 0.0 ;    IT2 ← 5.0 × G ;
LB9:
          P ← 0.99999999 ;
          IT1 ← 0.02 ;
          M ← 0 ;
          N1 ← 0 ;
          QPM1 ← 0.0 ;
LB2:
          IF (P>1) OR (P<0.0)  THEN
BEGIN     WRITE(PRINT,NOTE2) ;
```

```
          GO TO JOB ;                                                  END ;
          THBR ← APAR - THAR ;
          TP1 ← P×SIN(THAR/2) ;
          PHAR ← ARCTAN(TP1/ SQRT(1 -TP1*2)) ;
          TP1 ← P×SIN(THBR/2) ;
          PHBR ← ARCTAN(TP1/ SQRT(1 -TP1*2)) ;
          TP1 ← THAR ;
          C1 ← THAR/2 ;
          C2 ← THBR/2 ;
          TPS ← COS(PHBR) - COS(PHAR) ;
          TPC ← TPS/ (E(P,C1) +E(P,C2) -(1-P*2/2)×(F(P,C1) +F(P,C2))) ;
          IF ABS(TPC ) < 1.0@-10  THEN
LB22:     IF M = 0 THEN
BEGIN     N1 ← 0 ;
          IT1 ← 0.02 ;
          QPM1 ← 0.0 ;
          P ← P - 0.010 ;
          M ← 1 ;    GO TO LB2 ;                                      END
          ELSE
BEGIN     WRITE(PRINT,NOTE2) ;      GO TO JOB ;                       END ;
          TA ← COS(THBR)/TPC -SIN(THBR) ;
          IF ABS(TA  ) < 1.0@-10  THEN
          GO TO LB22 ;
          DD← (R×(COS(TP1)/TPC +SIN(TP1)))/( COS(THBR)/TPC -SIN(THBR )) ;
          Q1 ← DD - D ;
IF ABS(Q1)>1.0@-6THEN BEGIN IF N1<100THEN BEGIN IF(QPM1≠0.0)AND(SIGN(Q1)
-SIGN(QPM1)≠0.0)THEN IT1←-IT1/2.0;P←P-IT1;IF P<0.2THEN BEGIN WRITE(PRINT
,NOTE1);GO TO JOB;END;QPM1←Q1;N1←N1+1;GO TO LB2;END ELSE BEGIN WRITE(PRI
NT,NOTE3);GO TO JOB;END;END;
          K ← (2/P)×TPS/(-R×COS(TP1) +D×COS(THBR)) ;
          EL ← 2 ;
          H ← 2/(P×K) ;
          L ← (P/K)×(F(P,C1) + F(P,C2)) ;
          Q2 ← L - LL ;
IF ABS(Q2)>0.000001THEN BEGIN IF N2<100THEN BEGIN IF(Q2>0)THEN CJ←-1ELSE
 CJ←1;IF(QPM2≠0)AND((SIGN(Q2)-SIGN(QPM2))≠0.0)THEN IT2←IT2/2;THAR←THAR+C
J×IT2;THA←THAR/G;QPM2←Q2;N2←N2+1;GO TO LB8;END;END;WRITE(PRINT[PAGE]);WR
```

```
ITE(PRINT,TL2,AN2));
        L1 ← 2×L + 2×R×(PI-APAR) ;
LB4:
        PHA ← PHAR / G ;
        PHB ← PHBR / G ;
        WRITE(PRINT,TL3,AN3) ;
        IF EL = 1 THEN
        WRITE(PRINT,TL01)
        ELSE
        WRITE(PRINT,TL02) ;
JOB:

%                              CONFIGURATION (E)

     P = P, K = K, PSI M = THC, PSI N = THA
        R ←0.5 ;
        FOR APA ←  50.0 DO
        FOR D ← 0.75 DO
        FOR L1 ← 3.477 DO
BEGIN   APAR ← APA × G ;
        WRITE(PRINT [PAGE] ) ;
        WRITE(PRINT,TL2,AN2) ;
%CALCULATIONS FOR THE UNDULATING ELASTICA
        THA ←  10.0 ;    THAR ← THA × G ;
        IT2 ← 5.0 × G ;      QPM2 ← 0.0 ;
        LL ← (L1 - 2×R ×(PI - APAR))/ 2.0 ;
LB8:
        THCR ← APAR + THAR ;
        IF THCR > PI THEN
        GO TO JOB ;
        P ← 0.99999999 ;
        IT1 ← 0.02 ;
        M ← 0 ;
        N1 ← 0 ;
        QPM1 ← 0.0 ;
LB2:
        IF (P>1) OR (P<0.0)  THEN
```

```
BEGIN   WRITE(PRINT,NOTE2) ;     GO TO JOB ;                 END ;
        TP1 ← SIN(THCR/2 ) ;                                          0460
        IF TP1 > P THEN                                               0370
BEGIN   WRITE(PRINT,NOTE2) ;
        GO TO JOB ;                                          END ;
        PHCR ← ARCTAN(TP1/ SQRT(P*2 -TP1*2)) ;                        0470
        TP1 ← SIN(THAR/2 ) ;                                          0360
        IF TP1 > P THEN
BEGIN   WRITE(PRINT,NOTE2) ;
        GO TO JOB ;                                          END ;
        PHAR ← ARCTAN(TP1/ SQRT(P*2 -TP1*2)) ;                        0450
        TPS ← 2×P×(COS(PHAR) + COS(PHCR)) ;
        TP1 ← 2×(2×EC(P) -E(P,PHAR) -E(P,PHCR)) -2×KC(P)
             +F(P,PHAR) +F(P,PHCR) ;
        TPC ← TPS/TP1 ;
        IF ABS(TPC ) < 1.0@-10  THEN
LB22:   IF M = 0 THEN
BEGIN   N1 ← 0 ;
        IT1 ← 0.02 ;
        QPM1 ← 0.0 ;
        P ← P - 0.01 ;
        M ← 1 ;   GO TO LB2 ;                                END
        ELSE
BEGIN   WRITE(PRINT,NOTE2) ;      GO TO JOB ;                END ;
        TA ← SIN(THAR) +COS(THAR)/TPC ;
        IF ABS(TA  ) < 1.0@-10  THEN
        GO TO LB22 ;
        DD ← R×(SIN(THCR) + COS(THCR)/ TPC) / (COS(THAR)/TPC
            +SIN(THAR)) ;
        Q1 ← DD - D ;
IF ABS(Q1)>1.0@-6THEN BEGIN IF N1<100THEN BEGIN IF(QPM1≠0.0)AND(SIGN(Q1)00012700
-SIGN(QPM1)≠0.0)THEN IT1←-IT1/2.0;P←P-IT1;IF P<0.2THEN BEGIN WRITE(PRINT00012800
,NOTE1);GO TO JOB;END;QPM1←Q1;N1←N1+1;GO TO LB2;END ELSE BEGIN WRITE(PRI00012900
NT,NOTE3);GO TO JOB;END;END;                                          00013000
        EL ← 1 ;                                                      0680
        K ← TPS / (D ×COS(THAR) - R×COS(THCR) ) ;
        H ← 2×P/K ;
```

```
        L ← (2×KC(P) −F(P,PHAR) −F(P,PHCR))/ K ;
        Q2 ← L − LL ;
IF ABS(Q2)>0.000001THEN BEGIN IF N2<100THEN BEGIN IF(Q2>0)THEN CJ←−1ELSE
 CJ←1;IF(QPM2≠0)AND((SIGN(Q2)−SIGN(QPM2))≠0.0)THEN IT2←IT2/2;THAR←THAR−C
J×IT2;THA←THAR/G;QPM2←Q2;N2←N2+1;GO TO LBB;END;END;WRITE(PRINT[PAGE]);WR
ITE(PRINT,TL2,AN2));

        WRITE(PRINT [PAGE] ) ;
        WRITE(PRINT,TL2,AN2) ;
        L1 ← 2×L + 2×R×(PI−APAR) ;
        PHBR ← PI/ 2 ;
        PHA ← PHAR / G ;
        PHB ← PHBR / G ;
        PHC ← PHCR / G ;
        WRITE(PRINT,TL3,AN3) ;
        IF EL = 1 THEN
        WRITE(PRINT,TL01)
        ELSE
        WRITE(PRINT,TL02) ;
JOB:

%                                         CONFIGURATION (F)

    P = P, K = K, PSI M = THC, PSI N = THA
        R ←0.5 ;
        FOR D ← 0.75 DO
        FOR D ← 0.75 DO
BEGIN   APAR ← APA × G ;
        WRITE(PRINT [PAGE] ) ;
        WRITE(PRINT,TL2,AN2) ;
% SOLVE FOR  P AND K BY TRIAL AND ERROR
        QPM ← 0.0 ;
        P ← 0.99999999 ;
        IT ← 0.02 ;
        M ← 0 ;
        N ← 0 ;
        TPS ← SIN(APAR / 2.0) ;
        TP1 ← D − R × COS(APAR) ;
```

```
LB1:     IF (P×P ) < (TPS×TPS) THEN
         IF M = 0 THEN
BEGIN    PHD ← PI/2 ;
         P ← TPS ;
         M ← 1 ;                                               END
         ELSE
BEGIN    WRITE(PRINT,NOTE2) ;
         GO TO JOB ;                                           END
         ELSE PHD ← ARCTAN(TPS / SQRT(P×P - TPS×TPS)) ;
         K ← ((2.0×P)×(1.0 + COS(PHD)))/ TP1 ;
         Q ← (4.0×EC(P) - 2.0×KC(P)-2.0×E(P,PHD) +F(P,PHD))/ K
              -R×SIN(APAR) ;
         IF ABS(Q ) > 1.0@-5  THEN
BEGIN    IF N < 100 THEN
BEGIN    IF (QPM ≠ 0.0 ) AND (SIGN(Q)-SIGN(QPM) ≠ 0.0) THEN
         IT ← -IT / 2.0 ;
         P ← P - IT ;
         QPM ← Q ;
         N ← N + 1 ;
         GO TO LB1 ;                                           END
         ELSE
BEGIN    WRITE(PRINT, NOTE3) ;
        GO TO JOB ;                                            END ; END ;
         EL ← 1 ;
         H ← (2.0×P ) / K ;
         L ← (2×KC(P) - F(P,PHD))/ K ;
         L1 ← 2×L +2×R×(PI -APAR) ;
         PHID ← PHD / G ;
         GAMR ← ARCTAN(P/ SQRT(1.0-P×P)) ;
         GAM ← GAMR / G ;
         WRITE(PRINT,TL3,AN3) ;
         IF EL = 1 THEN
         WRITE(PRINT,TL01)
         ELSE
         WRITE(PRINT,TL02) ;
JOB:
```

```
%                             CONFIGURATION (G)
      P = P, K = K, PSI M = THC, PSI N = THA
         R ←0.5 ;
         FOR APA ←  50.0 DO
         FOR D ← 0.75 DO
         FOR L1 ← 3.500 DO
BEGIN    APAR ← APA ×  G;
         WRITE(PRINT [PAGE] ) ;                                  0810
         WRITE(PRINT,TL2,AN2) ;                                  0820
%CALCULATIONS FOR THE UNDULATING ELASTICA
         LL ← (L1 − 2×R ×(PI − APAR))/ 2.0 ;
         THA ←  10.0 ;   THAR ← THA × G ;
         QPM2 ← 0.0 ;    IT2 ← 5.0 × G ;
LB8:
         THCR ← APAR − THAR ;
         IF THCR > PI THEN
         GO TO JOB ;
         P ← 0.99999999 ;
         IT1 ← 0.02 ;
         M ← 0 ;
         N1 ← 0 ;                                                0230
         QPM1 ← 0.0 ;                                            0240
LB2:                                                             0250
         IF (P>1) OR (P<0.0)  THEN                               026
BEGIN    WRITE(PRINT,NOTE2) ;    GO TO JOB ;          END ;
         TP1 ← SIN(THCR/2 ) ;
         IF TP1 > P THEN                                         0460
BEGIN    WRITE(PRINT,NOTE2) ;                                    0370
         GO TO JOB ;                                  END ;
         PHCR ← ARCTAN(TP1/ SQRT(P*2 −TP1*2)) ;
         TP1 ← SIN(THAR/2 ) ;                                    0470
         IF TP1 > P THEN                                         0360
BEGIN    WRITE(PRINT,NOTE2) ;
         GO TO JOB ;                                  END ;
         PHAR ← ARCTAN(TP1/ SQRT(P*2 −TP1*2)) ;
         TPS ← 2×P×(COS(PHAR) + COS(PHCR)) ;                     0450
```

```
           TP1 ← 2×(2×EC(P) +E(P,PHAR) −E(P,PHCR)) −2×KC(P)
                 −F(P,PHAR) +F(P,PHCR) ;
           TPC ← TPS/TP1 ;
           IF ABS(TPC ) < 1.0@−10  THEN
LB22:      IF M = 0 THEN
BEGIN      N1 ← 0 ;
           IT1 ← 0.02 ;
           QPM1 ← 0.0 ;
           P ← P − 0.01 ;
           M ← 1 ;   GO TO LB2 ;                             END
           ELSE
BEGIN      WRITE(PRINT,NOTE2) ;      GO TO JOB ;              END ;
           TA ← −SIN(THAR) +COS(THAR)/TPC  ;
           IF ABS(TA  ) < 1.0@−10  THEN
           GO TO LB22 ;
           DD ← R×(SIN(THCR) + COS(THCR)/ TPC) / TA ;
           Q1 ← DD − D ;
IF ABS(Q1)>1.0@−6THEN BEGIN IF N1<100THEN BEGIN IF(QPM1≠0.0)AND(SIGN(Q1)
−SIGN(QPM1)≠0.0)THEN IT1←−IT1/2.0;P←P−IT1;IF P<0.2THEN BEGIN WRITE(PRINT
,NOTE1);GO TO JOB;END;QPM1←Q1;N1←N1+1;GO TO LB2;END ELSE BEGIN WRITE(PRI
NT,NOTE3);GO TO JOB;END;END;
           EL ← 1 ;
           K ← TPS / (D ×COS(THAR) − R×COS(THCR) ) ;
          H ← 2 × P / K ;
           L ← (2×KC(P) + F(P,PHAR) − F(P,PHCR)) / K ;
           Q2 ← L − LL ;
IF ABS(Q2)>0.000001THEN BEGIN IF N2<100THEN BEGIN IF(Q2>0)THEN CJ←−1ELSE
 CJ←1;IF(QPM2≠0)AND((SIGN(Q2)−SIGN(QPM2))≠0.0)THEN IT2←IT2/2;THAR←THAR+C
J×IT2;THA←THAR/G;QPM2←Q2;N2←N2+1;GO TO LB8;END;END;WRITE(PRINT[PAGE]);WR
ITE(PRINT,TL2,AN2);
           WRITE(PRINT [PAGE] ) ;
           WRITE(PRINT,TL2,AN2) ;
           L1 ← 2×L + 2×R×(PI−APAR) ;
           PHBR ← PI/ 2 ;
           PHA ← PHAR / G ;
           PHB ← PHBR / G ;
           PHC ← PHCR / G ;
```

```
            WRITE(PRINT,TL3,AN3) ;
            IF EL = 1 THEN
            WRITE(PRINT,TL01)
            ELSE
            WRITE(PRINT,TL02) ;
JOB:

* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *

DESCRIBING THE CAM CURVE BY CALCULATING A SERIES OF COORDINATE POINTS ON
THE CAM CURVE AND TRANSFORMING THESE TO THE (RS,TH) COORDINATE SYSTEM

DEFINE  SEQ1 =
BEGIN   X[I] ← (2.0×E(P,PHR) − F(P,PHR))/ K ;
        Y[I] ← H× COS(PHR) ;                                  END # ;
DEFINE  SEQ2 =
BEGIN   C1 ← SIN(PHR) ;
        C1 ← ARCTAN(C1/ SQRT(P*2 − C1*2)) ;
        X[I] ← H×(E(P,C1) − (1−P*2/2) ×F(P,C1)) ;
        Y[I] ← H× COS(PHR) ;                                  END # ;
DEFINE SEQ3 =
BEGIN   C1×((YM−C2×Y[I])×TPC + (XM−C4−C3×X[I])×TPS + R ;
        C1×((YM−C2×Y[I])×TPS − (XM−C4−C3×X[I])×TPC ;
        IF ABS(U[I]) < 1.0@−6  THEN THR[I] ←  PI/2
        ELSE THR[I] ← ARCTAN(V[I] / U[I] ) ;
        IF U[I] < 0.0 THEN THR[I] ← THR[I] + PI ;
        IF THR[I] < 0.0 THEN THR[I] ← THR[I] + 2.0×PI ;
       IF ABS(THR[I] − 3×PI/2) < 0.001 THEN THR[I] ← PI/2 ;
        RS[I] ← SQRT(U[I]×U[I] + V[I]×V[I] ) ;
        IF RS[I] < (R−0.001) THEN
BEGIN   WRITE(PRINT,NOTE4) ;
        IF EL = 1 THEN
        GO TO LB3 ;
        GO TO JOB ;                                           END ;
        TH[I] ← THR[I] / G ;
        I ← I + 1 ;                                           END # ;
```

```
          IF EL = 1 THEN
BEGIN     XM ← (2×E(P,PHMR) -F(P,PHMR))/K ;
          XC ← (2×E(P,PHCR) -F(P,PHCR))/K ;
          YM ← H × COS(PHAR) ;                                   END
          ELSE
BEGIN     XM ← H× (E(P,THMR/2) - (1-P*2/2)× F(P,THMR/2)) ;
          XC ← H× (E(P,THCR/2) - (1-P*2/2)× F(P,THCR/2)) ;
          YM ← H × COS(PHMR) ;                                   END ;
          THR[0] ← -0.02 ;
          RS[0] ← R ;
          I ← 1 ;                                                          0890
          TPS ← SIN(THMR) ;
          TPC ← COS(THMR) ;

THE PROGRESSIONS OF PHI FROM PHI M TO PHI N FOR DIFFERENT CONFIGURATIONS

FOR CONFIGURATIONS (A), (E), AND (F)
          CONF. (A) ;    C1 ← 1 ;         CONF. (E) AND (F) ;  C1 ← -1 ;
          FOR PHR ← PHMR STEP  0.02 UNTIL (PI/2) DO
BEGIN     IF EL = 1 THEN SEQ1 ELSE SEQ2 ;
          C2 ← 1 ;        C3 ← 1 ;      C4 ← 0.0 ;
          SEQ3 ;                                                 END ;
          FOR PHR ← (PI/2) STEP -0.02 UNTIL PHNR, PHNR DO
BEGIN     IF EL = 1 THEN SEQ1 ELSE SEQ2 ;
          CONF. (A) ;    C2 ← 1 ;         CONF. (E) AND (F) ;  C2 ← -1 ;
          C3 ← -1 ;        C4 ← 2×XC ;
          SEQ3 ;                                                 END ;

FOR CONFIGURATIONS (B) AND (C)
          C1 ← -1 ;  C2 ← 1 ;      C3 ← 1 ;      C4 ← 0.0 ;
          FOR PHR ← PHMR STEP -0.02 UNTIL PHNR, PHNR DO
BEGIN     IF EL = 1 THEN SEQ1 ELSE SEQ2 ;
          SEQ3 ;                                                 END ;

FOR CONFIGURATION  (D)
          C1 ← -1 ;        C2 ← 1 ;        C4 ← 0.0 ;
          FOR PHR ← PHMR STEP -0.02 UNTIL 0.0 DO
```

```
BEGIN    IF EL = 1 THEN SEQ1 ELSE SEQ2 ;
         C3 ← 1 ;
         SEQ3 ;                                                   END ;
         FOR PHR ← 0.0  STEP  0.02 UNTIL PHNR, PHNR DO
BEGIN    IF EL = 1 THEN SEQ1 ELSE SEQ2 ;
         C3 ← -1 ;
         SEQ3 ;                                                   END ;

 FOR CONFIGURATION  (G)
         C1 ← 1 ;
         FOR PHR ← PHMR STEP  0.02 UNTIL (PI/2) DO
BEGIN    SEQ2 ;
         C2 ← 1 ;     C3 ← 1 ;      C4 ← 0.0 ;
          SEQ3 ;                                                  END ;         1200
         FOR PHR ← (PI/2) STEP -0.02 UNTIL 0.0 DO
BEGIN    SEQ1 ;
         C2 ← -1 ;       C3 ← -1 ;        C4 ← 2 ×XC ;
          SEQ3 ;                                                  END ;         1040
         FOR PHR ← 0.0  STEP  0.02 UNTIL PHNR, PHNR DO
BEGIN    SEQ1 ;
         C2 ← -1 ;        C3 ← 1 ;        C4 ← 2×XC ;
         SEQ3 ;                                                   END ;

         TH[I] ← THR[I] ← 0.0 ;
         THR[I] ← 2.0×APAR - THR[I-2] ;
         TH[I] ← THR[I]/G ;
         RS[I] ← RS[I-2] ;
         I ← I - 1 ;

* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *

 PROCEDURES FOR CALCULATING THE FOLLOWER PATH FROM THE CAM PROFILE

%FOLLOWER PATH FOR TRANSLATING ROLLER FOLLOWER SYSTEM
AR←0.000;RR←0.375;FOR J←1STEP 1UNTIL I DO BEGIN C1←THR[J+1]-THR[J];C2←TH
R[J]-THR[J-1];APAR←ARCTAN((RS[J+1]×COS(C1)-RS[J-1]×COS(C2))/(RS[J+1]×SIN
(C1)+RS[J-1]×SIN(C2)));APAR←PI/2-APAR;GG←RS[J]*2+RR*2+2×RR×RS[J]×SIN(APA
```

```
R);X[J]←SQRT(GG-AR*2);TP1←RR×COS(APAR);U[J]←THR[J]-ARCTAN(AR/X[J])-ARCTA
N(TP1/SQRT(GG-TP1*2));V[J]←X[J];END;U[0]←U[I-2];X[0]←X[I-2];V[0]←X[0];WR
ITE(PRINT,TL220);

%FOLLOWER PATH FOR TRANSLATING FLAT-FACED FOLLOWER SYSTEM
FOR J←1STEP 1UNTIL I DO BEGIN C1←THR[J+1]-THR[J];C2←THR[J]-THR[J-1];APAR
←ARCTAN((RS[J+1]×COS(C1)-RS[J-1]×COS(C2))/(RS[J+1]×SIN(C1)+RS[J-1]×SIN(C
2)));U[J]←THR[J]-APAR;V[J]←RS[J]×COS(APAR);X[J]←V[J];END;U[0]←U[I-2];V[0
]←V[I-2];WRITE(PRINT,TL24);

%FOLLOWER PATH FOR ROLLER FOLLOWER ON RADIAL ARM
AR←1.000;BR← 4.00004;CR← 4.0;RR←0.375;
TP1← SQRT(ABS(CR*2-BR*2));                                 IF BR<CR THEN 00001400
IF TP1<AR THEN PPM←(AR-TP1)*2ELSE PPM←0.0ELSE PPM←400.0;FOR J←1STEP 1UNT00001500
IL I DO BEGIN C1←THR[J+1]-THR[J];C2←THR[J]-THR[J-1];APAR←ARCTAN((RS[J+1]00001600
×COS(C1)-RS[J-1]×COS(C2))/(RS[J+1]×SIN(C1)+RS[J-1]×SIN(C2)));APAR←PI/2-A00001700
PAR;GG←RS[J]*2+RR*2+2×RR×RS[J]×SIN(APAR);SA←AR*2+BR*2;K←SA-CR*2+GG;SB←-A00001800
R×K;SC←K*2/4-BR*2×GG;Z←(-SB+SQRT(SB*2-4×SA×SC))/(2×SA);IF(Z<0)OR((Z*2)>G00001900
G)THEN Z←(-SB-SQRT(SB*2-4×SA×SC))/(2×SA);H←SQRT(GG-Z*2);IF GG<PPM THEN H00002000
←-H;V[J]←ARCTAN((AR-Z)/(BR+H));TP1←RR×COS(APAR);U[J]←THR[J]-ARCTAN(TP1/S00002100
QRT(GG-TP1*2))+ARCTAN(H/Z);X[J]←Z;END;U[0]←-0.05;V[0]←ARCTAN((AR-R)/BR);00002200
X[0]←R+RR;WRITE(PRINT,TL21);
                                                                        00002300

%FOLLOWER PATH FOR OSCILLATING FLAT-FACED FOLLOWER
AR←1.025;BR←12.00;RR←0.375;FOR J←1STEP 1UNTIL I DO BEGIN C1←THR[J+1]-THR
[J];C2←THR[J]-THR[J-1];APAR←ARCTAN((RS[J+1]×COS(C1)-RS[J-1]×COS(C2))/(RS
[J+1]×SIN(C1)+RS[J-1]×SIN(C2)));APAR←PI/2-APAR;CR←AR*2+BR*2;TP1←RR+RS[J]
×SIN(APAR);V[J]←ARCTAN(AR/BR)-ARCTAN(TP1/SQRT(CR-TP1*2));X[J]←0.0;U[J]←T
HR[J]+APAR+V[J]-PI/2;END;U[0]←U[I-2];V[0]←V[I-2];WRITE(PRINT,TL23);

* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *

 PROCEDURES TO NUMERICALLY FIND THE VELOCITY AND ACCELERATION CURVES
 OF THE CAM PROFILE AND THE FOLLOWER PATHS.  ALSO THE VALUES OF THE
 MAXIMUM ACCELERATIONS ARE FOUND.

WRITE(PRINT,TL12,AN12) ;  APAR ← APA×G ;                               00000000
```

```
ACPM←0.0;MNA←0.0;MPA←0.0;MJK←0.0;FOR N←1STEP 1UNTIL I DO BEGIN
 VE←(RS[N+1]-RS[N-1])/(THR[N+1]-THR[N-1]);ACPM←AC;AC←((2.0)×((RS[N+1]-RS
[N])/(THR[N+1]-THR[N])-((RS[N]-RS[N-1])/(THR[N]-THR[N-1]))))/(THR[N+1]-TH
R[N-1]));VEL←((2.0)×((X[N+1]-X[N])/(U[N+1]-U[N])-((X[N]-X[N-1])/(U[N]-U[
N-1]))))/(U[N+1]-U[N-1]));ACC←((2.0)×((V[N+1]-V[N])/(U[N+1]-U[N])-((V[N]-
V[N-1])/(U[N]-U[N-1]))))/(U[N+1]-U[N-1]));TPS←U[N]/G;TPC←V[N]/G;IF AC<MNA
 THEN MNA←AC;IF AC>MPA THEN MPA←AC;JK←(AC-ACPM)/(THR[N-1]-THR[N]);IF ABS
(JK)>ABS(MJK)THEN MJK←JK;Y[N]←VEL;WRITE(PRINT,TL13,AN13);END;WRITE(PRINT
,TL6,AN6);
BB←APAR;MAH←((D-R)/2.0)×(PI/BB)*2;MJH←-MAH×(PI/BB);MAC←(2×(D-R)×PI)/BB*2
;MJC←4.0×PI*2×(D-R)/(BB*3);WRITE(PRINT,TL10,AN10);
```

* *

PROCEDURES TO CALCULATE THE CONSTITUENT HARMONICS OF THE FOLLOWER ACCELERATION CURVE

```
FOR N ←0 STEP 1 UNTIL (S+2) DO  THR[N] ← U[N] ;
BEGIN REAL DUMMY;LABEL LB7;I←0;J←0;FOR Q1←0.0STEP 2.50UNTIL APA DO BEGIN
 Q←Q1×G;LB7:IF(Q>THR[I])AND(Q≤THR[I+1])THEN BEGIN U[J]←Q;V[J]←(Q-THR[I])
×( Y[I+1]- Y[I])/(THR[I+1]-THR[I])+ Y[I];S←J;J←J+1;END ELSE BEGIN I←I+1;
GO TO LB7;END;END;FOR I←1STEP 1UNTIL S DO BEGIN U[J]←2×Q-U[S-I];V[J]←V[S
-I];J←J+1;END;A[0]←0.0;FOR I←0STEP 1UNTIL(2×S-1)DO A[0]←A[0]+V[I];A[0]←A
[0]/(2×S);FOR J←1STEP 1UNTIL 2000 BEGIN A[J]←0.0;B[J]←0.0;FOR I←0STEP 1U
NTIL(2×S-1)DO BEGIN X[I]←PI×U[I]/APAR;A[J]←A[J]+V[I]×COS(J×X[I]);B[J]←B[
J]+V[I]×SIN(J×X[I]);END;A[J]←A[J]/S;B[J]←B[J]/S;END;WRITE(PRINT,TL11,AN1
1);END;
Q←PI/8.0;FOR TP1←Q STEP Q UNTIL PI DO BEGIN TA←(APAR×TP1)/(PI×G);C1←C2←0
.0;FOR J←1STEP 1UNTIL 2000 BEGIN C1←C1+A[J]×COS(J×TP1);C2←C2+B[J]×SIN(J×
TP1));END;CC←A[0]+C1+C2;WRITE(PRINT,TL99,AN99);END;
```

APPENDIX C

CALCULATION OF THE FOLLOWER PATHS FROM THE CAM PROFILE

The procedures which are used to calculate the follower paths from the cam profiles are presented in this Appendix. The notation for the types of followers will be:

r_i, Θ_i Polar coordinates of the points on the cam profile.

For Translating Followers

r_i', Θ_i' Polar coordinates of points on the follower path.

z Roller Follower Radius.

a Offset between follower center and cam center.

b Distance from cam center to follower center.

For Oscillating Followers:

ϕ_i', Θ_i' Coordinates of the points on the follower path, ϕ_i' is angle of rotation of the arm.

z Roller radius or distance from center line of the arm to the face of follower.

a Arm length

b Distance from cam center to follower center

c Vertical distance from cam center to follower pivot.

d Horizontal distance from cam center to follower pivot.

The cam profile is described by a series of coordinate points, r_i, Θ_i. Since these points are known, they will be used as the points of contact between the cam and the follower. The follower path is described in the series of points calculated from these. The velocity and acceleration of the follower are found by numerical differentiation of the follower displacement path.

Translating Roller Follower

To find the coordinates of the roller center when given the coordinates of the point of contact, the length r_i and the angle Θ_i must be calculated (Figure 58). The value of b is one side of the triangle OO'C. The angle at C of this triangle equals the angle γ, the angle the tangent to the cam profile makes with the line r_i, plus 90°. The value of γ is calculated at the point of contact $(r_i, \Theta_i{}')$ using the two adjacent coordinate points, (r_{i-1}, Θ_{i-1}) and (r_{i+1}, Θ_{i+1}) from the expression:

$$\gamma = 90 - \tan^{-1}\left[\frac{r_{i+1}\cos(\Theta_{i+1}-\Theta_i) - r_{i-1}\cos(\Theta_i-\Theta_{i-1})}{r_{i+1}\sin(\Theta_{i+1}-\Theta_i) + r_{i-1}\sin(\Theta_i-\Theta_{i-1})}\right] \qquad \text{(C-1)}$$

Using the law a cosines, b is found from:

$$b = \sqrt{r_i{}^2 + z^2 + 2\,zr_i\,\sin\gamma} \qquad \text{(C-2)}$$

Then the value of $r_i{}'$ is found from the right triangle AOO':

$$r_i{}' = \sqrt{a^2 + b^2} \qquad \text{(C-3)}$$

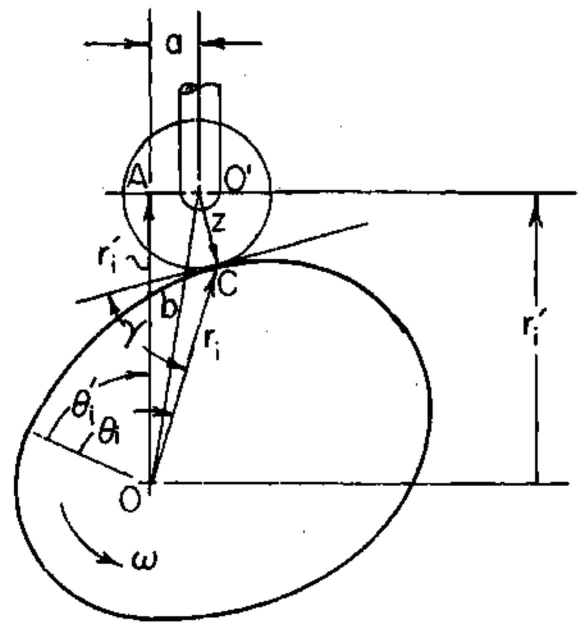


Figure 58. Translating Roller Follower

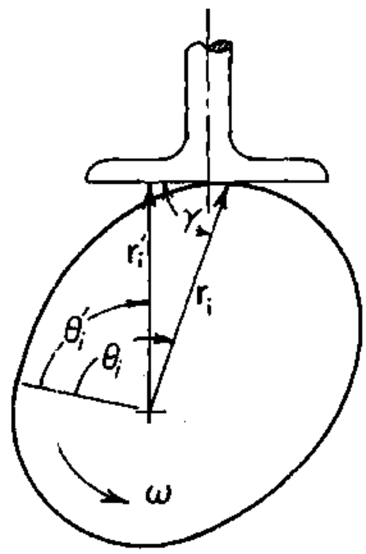


Figure 59. Translating Flat-Faced Follower

The value of θ_i' is found from:

$$\theta_i' = \theta_i - \angle AOO' - \angle O'OC$$

The angle AOO' is found from the triangle AOO' from:

$$\angle AOO' = \tan^{-1}\left[\frac{a}{r_i'}\right]$$

and from the law of sines for triangle AOO':

$$\angle O'OC = \tan^{-1}\left[\frac{z\cos\gamma}{\sqrt{b^2 - z^2\cos^2\gamma}}\right]$$

Hence:

$$\theta_i' = \theta_i - \tan^{-1}\left[\frac{a}{r_i'}\right] - \tan^{-1}\left[\frac{z\cos\gamma}{\sqrt{b^2 - z^2\cos^2\gamma}}\right] \qquad \text{(C-4)}$$

A series of precision points (r_i', θ_i') is calculated from the series of points (r_i, θ_i) representing the cam profile using the Equations (C-3) and (C-4).

Translating Flat-Faced Follower

The translating flat-faced follower system is shown in Figure 59. The follower path formed by the series of points (r_i', θ_i') is calculated from the series of points (r_i, θ_i) forming the cam profile. The angle r between the tangent to the cam curve at the point of contact and the line r_i, is calculated from Equation (C-1). Then r_i' is calculated from:

$$r_i' = r_i \sin\gamma \tag{C-5}$$

and Θ_i' is calculated from:

$$\Theta_i' = \Theta_i - (90^\circ - \gamma) \tag{C-6}$$

The series of precision points (r_i', Θ_i') defining the follower path are found from Equations (C-5) and (C-6).

Oscillating Roller Follower

For the oscillating roller follower system the value of ϕ_i', the angle of arm from the horizontal, and the angle Θ_i' must be calculated from the point of contact between the cam and the follower, (γ_i, Θ_i) (Figure 60). The angle γ is calculated from Equation (C-1). Then b is calculated from Equation (C-2).

From Figure 60 we find:

$$\sin\phi = \frac{c - f}{a}$$

$$\cos\phi = \frac{x + d}{a} \tag{C-7}$$

But

$$\sin^2\phi + \cos^2\phi = 1 \tag{C-8}$$

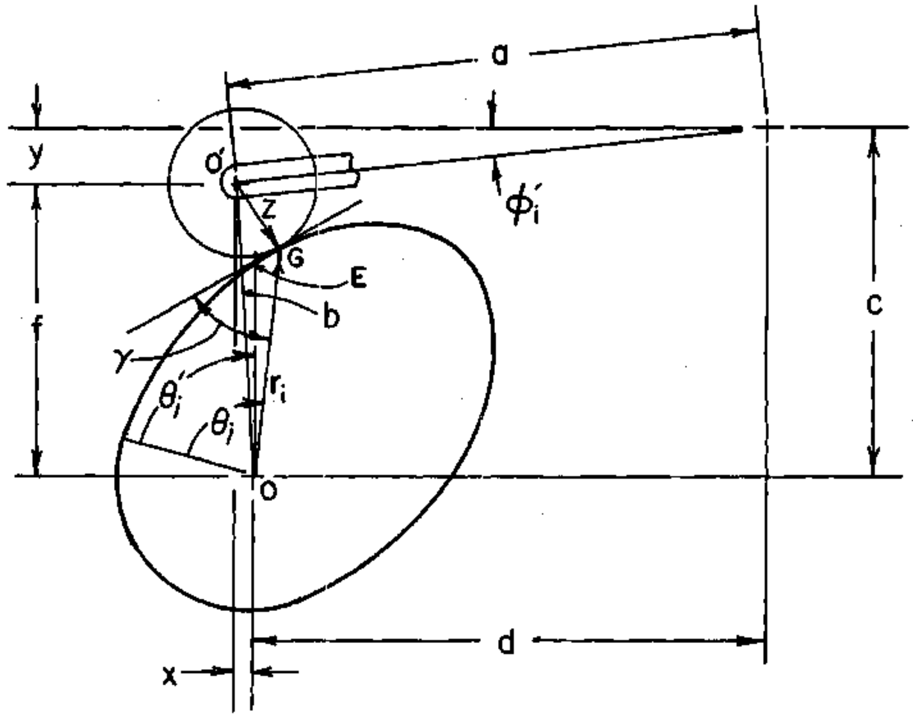


Figure 60. Oscillating Roller Follower

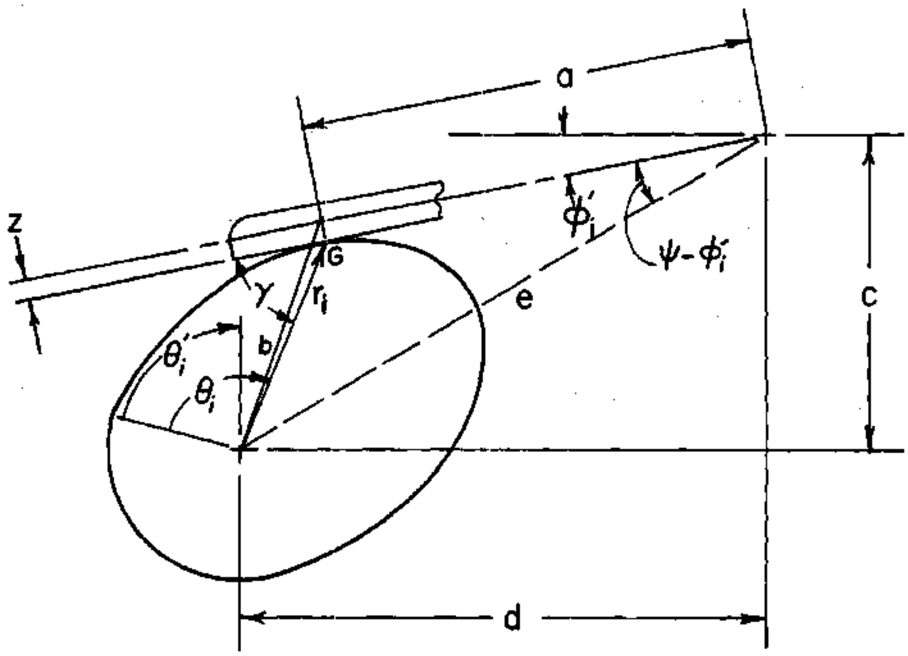


Figure 61. Oscillating Flat-Faced Follower

Therefore, from Equations (C-7) and (C-8) we find:

$$a^2 = (c - f)^2 + (x - d)^2 \tag{C-9}$$

From the right triangle with sides x, f, and b, we calculate the value of x:

$$x = \sqrt{b^2 - f^2} \tag{C-10}$$

Sbustituting Equation (C-10) into Equation (C-9) we obtain

$$f = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \tag{C-9}$$

where:

$$K = (c + d - a + b)$$

$$A = (c + d)$$

$$B = - c K$$

$$C = \frac{1}{4} K - d b$$

To determine the sign in Equation (C-11) the criterion that f must always be positive but less than or equal to b is used. Also, it must be determined if x will be positive or negative, that is, on which side of the cam center will the roller center lie. If d is greater than a, then x is always negative, as the roller center must lie inside the cam center. For other cases, we start by calculating the value of b when the roller center is directly over the cam center:

$$b' = c - \sqrt{a^2 - d^2} \tag{C-12}$$

Then the value of b at the point in question is calculated. If:

$b > b'$ then x is positive

$b = b'$ then $x = 0$

$b < b'$ then x is negative

The value of ϕ_i' is calculated from:

$$\phi_i' = \tan^{-1}\left[\frac{c - f}{d + x}\right] \tag{C-13}$$

where the values of f and x are found from Equations (C-11) and (C-10), respectively. The value of Θ_i' is found from:

$$\Theta_i' = \Theta_i + \angle O'OG - \angle O'OE \tag{C-14}$$

Substituting the equations for the angles O'OG and O'OE, Equation (C-14) becomes:

$$\Theta_i' = \Theta_i + \tan^{-1}\left[\frac{z\cos\gamma}{\sqrt{b^2 - z^2\cos^2\gamma}}\right] - \tan^{-1}\left[\frac{x}{f}\right] \qquad \text{(C-15)}$$

The series of precision points (ϕ_i', Θ_i') defining the follower path are found from Equations (C-13) and (C-15).

Oscillating Flat-Faced Follower

For the oscillating flat-faced follower system the coordinate points (ϕ_i', Θ_i') are established from the cam coordinate point (r_i, Θ_i) from the geometry shown in Figure 61. The values of γ and b are calculated from Equations (C-1) and (C-2), respectively. The angle ψ is calculated from:

$$\psi = \tan^{-1}\left(\frac{c}{d}\right) \qquad \text{(C-16)}$$

and the length e from:

$$e = \sqrt{c^2 + d^2} \qquad \text{(C-17)}$$

Now using the law of sines and equating the side b of the triangles e-a-b and r_i - z - b, we obtain the expression:

$$\sin(\psi - \phi_i) = \frac{z + r_i\sin\gamma}{e} \qquad \text{(C-18)}$$

from this equation and Equations (C-16), and (C-17), we obtain an expression for ϕ_i':

$$\phi_i' = \tan^{-1}\left(\frac{c}{d}\right) - \tan^{-1}\left[\frac{z + r_i \sin\gamma}{e^2 - (z + r_i \sin\gamma)^2}\right] \qquad \text{(C-19)}$$

Then to find Θ_i' we use:

$$\Theta_i' = \Theta_i - [90^\circ - (\gamma + \phi_i')]$$

This reduces to:

$$\Theta_i' = \Theta_i + \gamma + \phi_i - 90^\circ \qquad \text{(C-20)}$$

The series of precision points (ϕ_i', Θ_i') defining the follower path are found from Equations (C-19) and (C-20).

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VITA

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